Vectors

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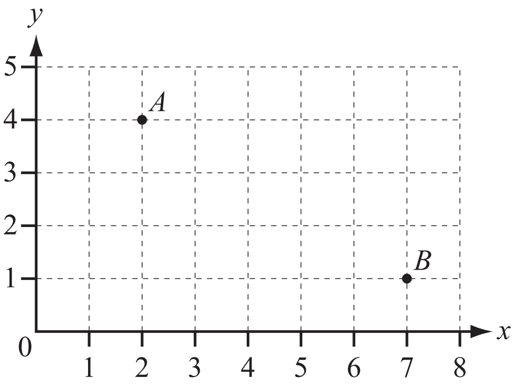
5 **(a)**



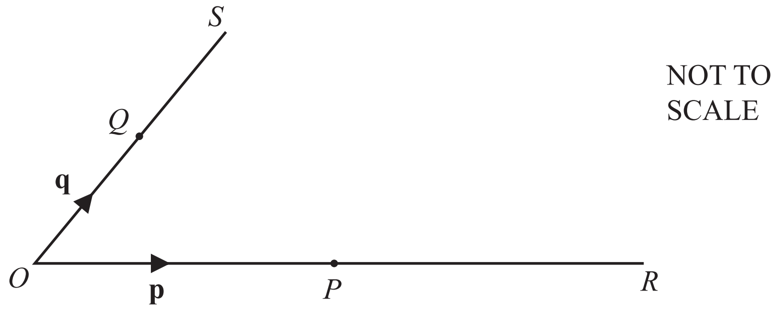
(i) Write down the position vector of A.

Answer(a)(i) $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$

(ii) Find $|\overrightarrow{AB}|$, the magnitude of $\overrightarrow{AB}$.

Answer(a)(ii) .... [2]

(b)



O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
OP is extended to R so that $OP = PR$.
OQ is extended to S so that $OQ = QS$.

(i) Write down $\overrightarrow{RQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer(b)(i) $\overrightarrow{RQ}$ = ... [1]

(ii) PS and RQ intersect at M and $RM = 2MQ$.

Use vectors to find the ratio $PM : PS$, showing all your working.

Answer(b)(ii) $PM : PS$ = ... : .... [4]

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NOT TO SCALE

A is the point (–1, 1) and B is the point (8, 7).

(a) Write $\overrightarrow{AB}$ as a column vector.

Answer(a) $\overrightarrow{AB} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

(b) Find $|\overrightarrow{AB}|$.

Answer(b) $|\overrightarrow{AB}| =$ .... [2]

(c) $\overrightarrow{AC} = 2\overrightarrow{AB}$.

Write down the co-ordinates of C.

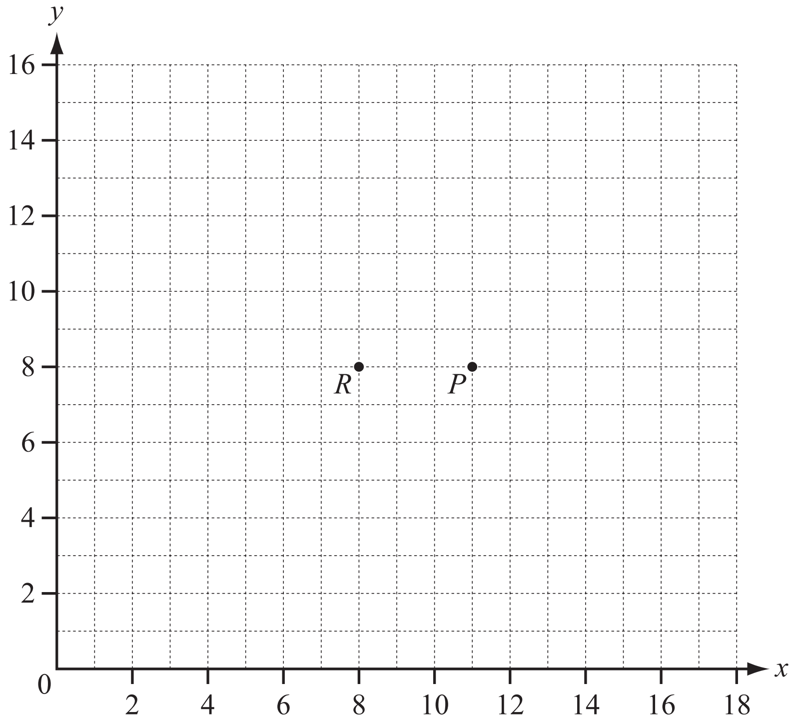
Answer(c) (................ , ...) [1]

For Examiner's Use

6 **(a)** Calculate the magnitude of the vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

Answer(a) .... [2]

(b)



(i) The points P and R are marked on the grid above.

$\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$. Draw the vector $\overrightarrow{PQ}$ on the grid above. [1]

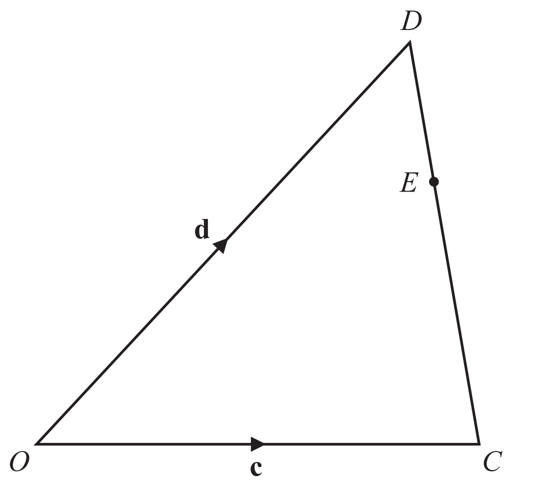
(ii) Draw the image of vector $\overrightarrow{PQ}$ after rotation by 90° anticlockwise about R. [2]

(c) $\overrightarrow{DE} = 2\mathbf{a} + \mathbf{b}$ and $\overrightarrow{DC} = 3\mathbf{b} - \mathbf{a}$.

Find $\overrightarrow{CE}$ in terms of **a** and **b**. Write your answer in its simplest form.

Answer(c) $\overrightarrow{CE}$ = ... [2]

20



In the diagram, O is the origin.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.
E is on CD so that $CE = 2ED$.

Find, in terms of **c** and **d**, in their simplest forms,

(a) $\overrightarrow{DE}$,

Answer(a) $\overrightarrow{DE} =$ .... [2]

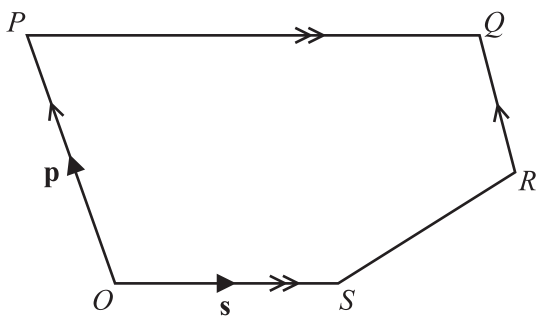
(b) the position vector of E.

Answer(b) .... [2]

For Examiner's Use

For Examiner's Use

(b)



NOT TO SCALE

In the pentagon *OPQRS*, *OP* is parallel to *RQ* and *OS* is parallel to *PQ*.
$PQ = 2OS$ and $OP = 2RQ$.
O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OS} = \mathbf{s}$.

Find, in terms of **p** and **s**, in their simplest form,

(i) the position vector of *Q*,

Answer(b)(i) .... [2]

(ii) $\overrightarrow{SR}$.

Answer(b)(ii) $\overrightarrow{SR}$ = ... [2]

(c) Explain what your answers in **part (b)** tell you about the lines *OQ* and *SR*.

Answer(c) .... [1]

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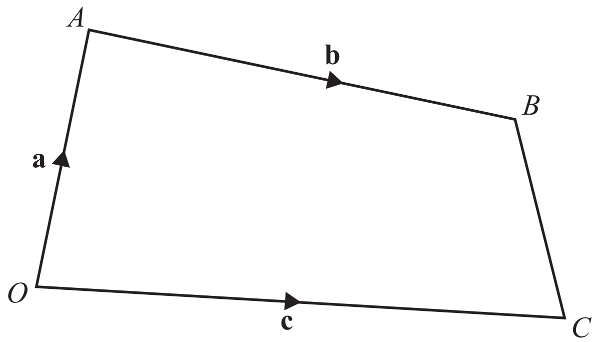


In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{AB} = \mathbf{b}$.
P is on the line AB so that $AP : PB = 2 : 1$.
Q is the midpoint of BC.

Find, in terms of **a**, **b** and **c**, in its simplest form

(a) $\overrightarrow{CB}$,

$\overrightarrow{CB}$ = ... [1]

(b) the position vector of Q,

.... [2]

(c) $\overrightarrow{PQ}$.

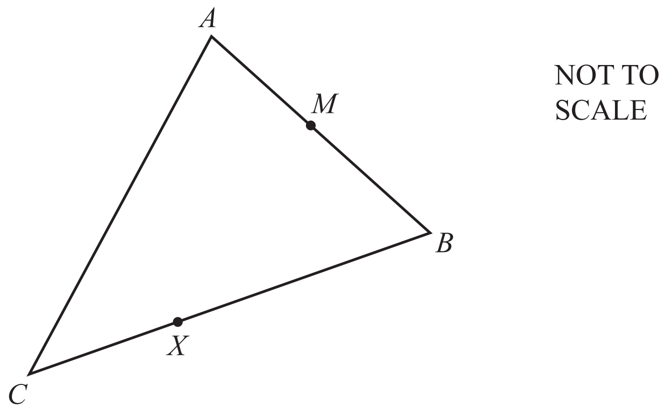
$\overrightarrow{PQ}$ = ... [2]

(d) $\overrightarrow{OT} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\overrightarrow{OV} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Write $\overrightarrow{TV}$ as a column vector.

Answer(d) $\overrightarrow{TV} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(e)



$\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$.

(i) Find $\overrightarrow{CB}$ in terms of **b** and **c**.

Answer(e)(i) $\overrightarrow{CB}$ = ... [1]

(ii) X divides CB in the ratio $1:3$.
M is the midpoint of AB.

Find $\overrightarrow{MX}$ in terms of **b** and **c**.
Show all your working and write your answer in its simplest form.

Answer(e)(ii) $\overrightarrow{MX}$ = ... [4]

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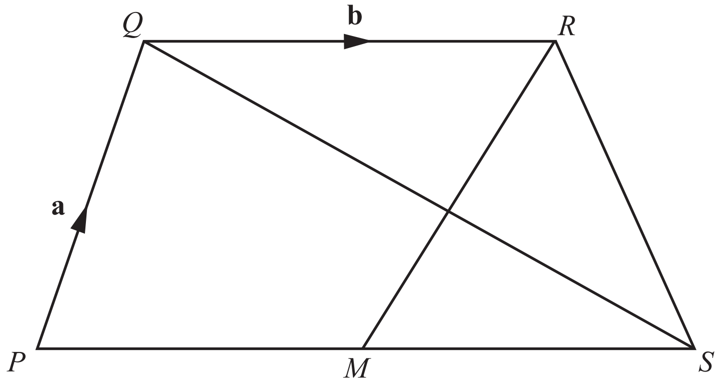


NOT TO SCALE

PQRS is a quadrilateral and *M* is the midpoint of *PS*.
$\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$ and $\overrightarrow{SQ} = \mathbf{a} - 2\mathbf{b}$.

(a) Show that $\overrightarrow{PS} = 2\mathbf{b}$.

Answer(a)

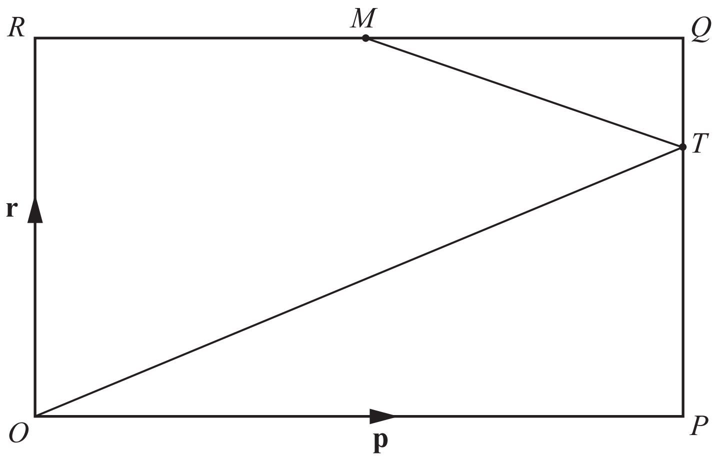
[1]

(b) Write down the mathematical name for the quadrilateral *PQRM*, giving reasons for your answer.

Answer(b) .... because ....

.... [2]

7



OPQR is a rectangle and *O* is the origin.
M is the midpoint of *RQ* and $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and/or **r**, in its simplest form

(i) $\overrightarrow{MQ}$,

$\overrightarrow{MQ} =$ .... [1]

(ii) $\overrightarrow{MT}$,

$\overrightarrow{MT} =$ .... [1]

(iii) $\overrightarrow{OT}$.

$\overrightarrow{OT} =$ .... [1]

(b) *RQ* and *OT* are extended to meet at *U*.

Find the position vector of *U* in terms of **p** and **r**.
Give your answer in its simplest form.

.... [2]

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

Find the positive value of k.

$k =$ .... [3]

9 **(a)** $\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

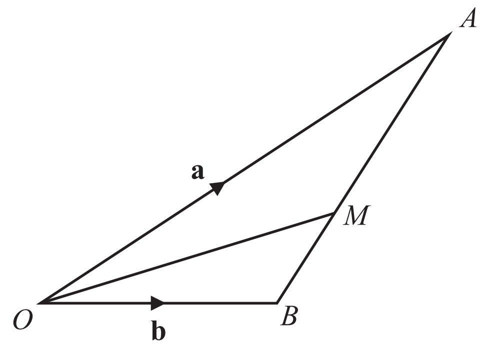
(i) Work out $2\mathbf{m} - 3\mathbf{n}$.

$\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(ii) Calculate $|2\mathbf{m} - 3\mathbf{n}|$.

.... [2]

(b) **(i)**



NOT TO SCALE

In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
The point M lies on AB such that $AM : MB = 3 : 2$.

Find, in terms of **a** and **b**, in its simplest form

(a) $\overrightarrow{AB}$,

$\overrightarrow{AB}$ = ... [1]

(b) $\overrightarrow{AM}$,

$\overrightarrow{AM}$ = ... [1]

(c) the position vector of M.

.... [2]

(ii) OM is extended to the point C.
The position vector of C is $\mathbf{a} + k\mathbf{b}$.

Find the value of k.

k = ... [1]

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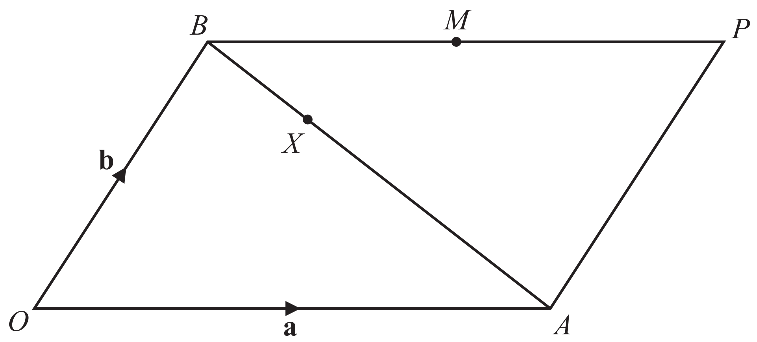


OAPB is a parallelogram.
O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is the midpoint of *BP*.

(a) Find, in terms of **a** and **b**, giving your answer in its simplest form,

(i) $\overrightarrow{BA}$,

Answer(a)(i) $\overrightarrow{BA}$ = ... [1]

(ii) the position vector of *M*.

Answer(a)(ii) .... [1]

(b) *X* is on *BA* so that $BX : XA = 1 : 2$.

Show that *X* lies on *OM*.

Answer(b)

[4]

Question 20 is printed on the next page.

10 (a) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$

(i) Find the value of $|\overrightarrow{PQ}|$.

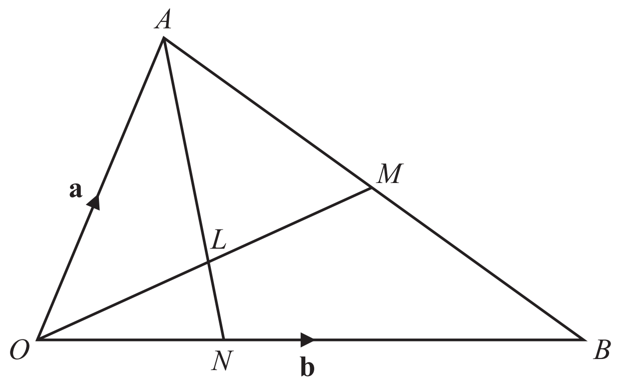
Answer(a)(i) $|\overrightarrow{PQ}|$ = ... [2]

(ii) Q is the point (2, –3).

Find the co-ordinates of the point P.

Answer(a)(ii) (..................... , ...) [1]

(b)



NOT TO SCALE

In the diagram, M is the midpoint of AB and L is the midpoint of OM.
The lines OM and AN intersect at L and $ON = \frac{1}{3}OB$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Find, in terms of **a** and **b**, in its simplest form,

(a) $\overrightarrow{OM}$,

Answer(b)(i)(a) $\overrightarrow{OM}$ = ... [2]

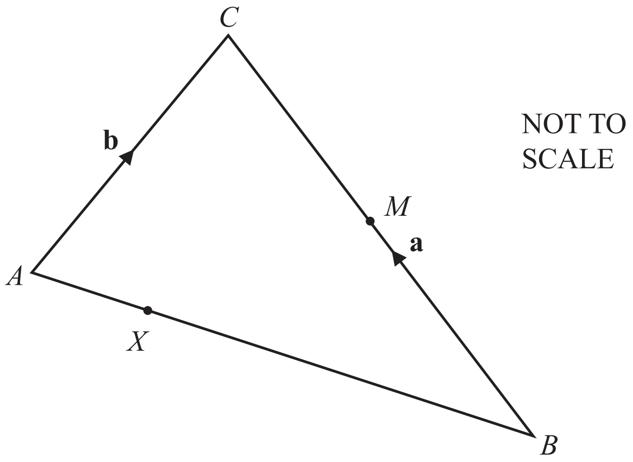
(b) $\overrightarrow{OL}$,

Answer(b)(i)(b) $\overrightarrow{OL}$ = ... [1]

(c) $\overrightarrow{AL}$.

Answer(b)(i)(c) $\overrightarrow{AL}$ = ... [2]

10



$\overrightarrow{BC} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

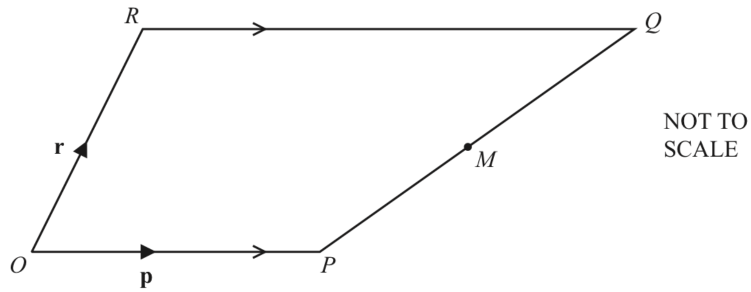
Answer(a) $\overrightarrow{AB}$ = ... [1]

(b) M is the midpoint of BC.
X divides AB in the ratio $1:4$.

Find $\overrightarrow{XM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Show all your working and write your answer in its simplest form.

Answer(b) $\overrightarrow{XM}$ = ... [4]

14



OPQR is a trapezium with *RQ* parallel to *OP* and $RQ = 2OP$.
O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.
M is the midpoint of *PQ*.

Find, in terms of **p** and **r**, in its simplest form

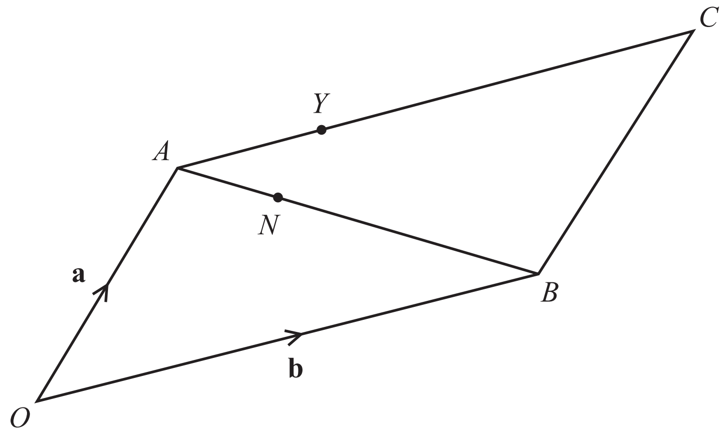
(a) $\overrightarrow{PQ}$,

Answer(a) $\overrightarrow{PQ}$ = ... [1]

(b) $\overrightarrow{OM}$, the position vector of *M*.

Answer(b) $\overrightarrow{OM}$ = ... [2]

(b)



OACB is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
$AN : NB = 2 : 3$ and $AY = \frac{2}{5}AC$.

(i) Write each of the following in terms of **a** and/or **b**.
Give your answers in their simplest form.

(a) $\overrightarrow{ON}$

Answer(b)(i)(a) $\overrightarrow{ON}$ = ... [2]

(b) $\overrightarrow{NY}$

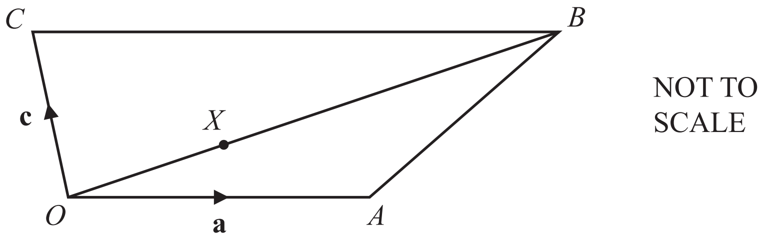
Answer(b)(i)(b) $\overrightarrow{NY}$ = ... [2]

(ii) Write down two conclusions you can make about the line segments *NY* and *BC*.

Answer(b)(ii) ....

.... [2]

19



The diagram shows a quadrilateral *OABC*.
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 2\mathbf{a}$.
X is a point on *OB* such that $OX : XB = 1 : 2$.

(a) Find, in terms of **a** and **c**, in its simplest form

(i) $\overrightarrow{AC}$,

Answer(a)(i) $\overrightarrow{AC}$ = ... [1]

(ii) $\overrightarrow{AX}$.

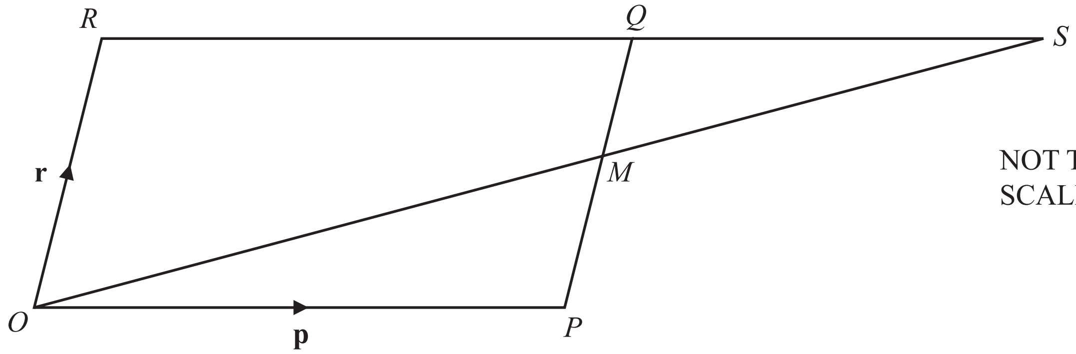
Answer(a)(ii) $\overrightarrow{AX}$ = ... [3]

(b) Explain why the vectors $\overrightarrow{AC}$ and $\overrightarrow{AX}$ show that *C*, *X* and *A* lie on a straight line.

Answer(b) ....

.... [2]

20



NOT TO SCALE

OPQR is a parallelogram, with *O* the origin.
M is the midpoint of *PQ*.
OM and *RQ* are extended to meet at *S*.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and **r**, in its simplest form,

(i) $\overrightarrow{OM}$,

Answer(a)(i) $\overrightarrow{OM} =$ .... [1]

(ii) the position vector of *S*.

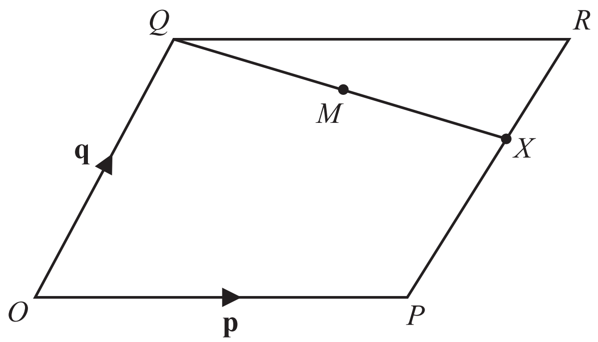
Answer(a)(ii) .... [1]

(b) When $\overrightarrow{PT} = -\frac{1}{2}\mathbf{p} + \mathbf{r}$, what can you write down about the position of *T*?

Answer(b) .... [1]

18

For Examiner's Use



NOT TO SCALE

O is the origin and $OPRQ$ is a parallelogram.
The position vectors of P and Q are $\mathbf{p}$ and $\mathbf{q}$.
X is on PR so that $PX = 2XR$.

Find, in terms of $\mathbf{p}$ and $\mathbf{q}$, in their simplest forms

(a) $\overrightarrow{QX}$,

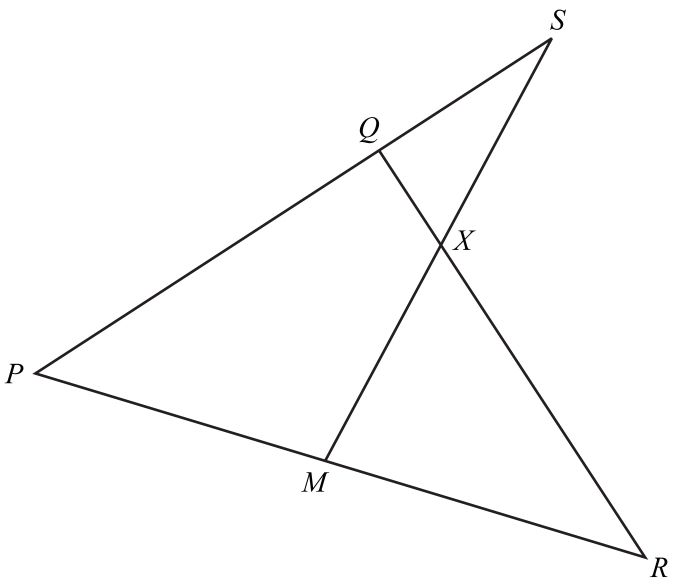
Answer(a) $\overrightarrow{QX}$ = ... [2]

(b) the position vector of M, the midpoint of QX.

Answer(b) .... [2]

For Examiner's Use

18



NOT TO SCALE

In the diagram, PQS, PMR, MXS and QXR are straight lines.

$PQ = 2\ QS$.
M is the midpoint of PR.
$QX : XR = 1 : 3$.

$\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PR} = \mathbf{r}$.

(a) Find, in terms of **q** and **r**,

(i) $\overrightarrow{RQ}$,

Answer(a)(i) $\overrightarrow{RQ} =$ .... [1]

(ii) $\overrightarrow{MS}$.

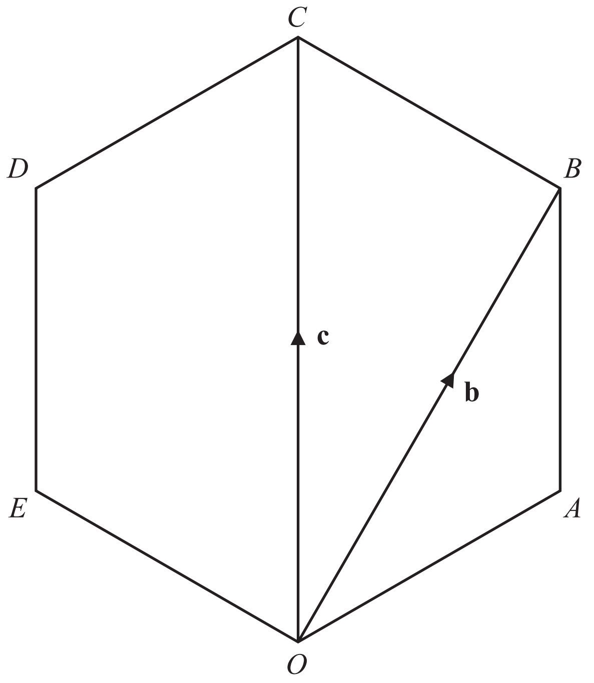
Answer(a)(ii) $\overrightarrow{MS} =$ .... [1]

(b) By finding $\overrightarrow{MX}$, show that X is the midpoint of MS.

Answer (b)

[3]

19



OABCDE is a regular polygon.

(a) Write down the geometrical name for this polygon.

Answer(a) .... [1]

(b) O is the origin. $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **b** and **c**, in their simplest form,

(i) $\overrightarrow{BC}$,

Answer(b)(i) $\overrightarrow{BC} =$ .... [1]

(ii) $\overrightarrow{OA}$,

Answer(b)(ii) $\overrightarrow{OA} =$ .... [2]

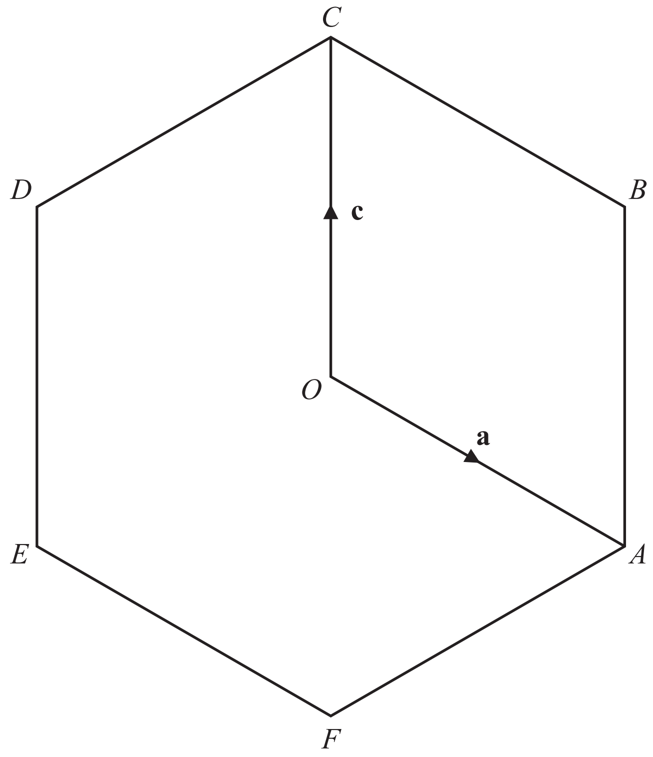
(iii) the position vector of E.

Answer(b)(iii) .... [1]

Question 20 is printed on the next page.

For Examiner's Use

19



O is the origin.
ABCDEF is a regular hexagon and *O* is the midpoint of *AD*.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **a** and **c**, in their simplest form

(a) $\overrightarrow{BE}$,

Answer(a) $\overrightarrow{BE}$ = ... [2]

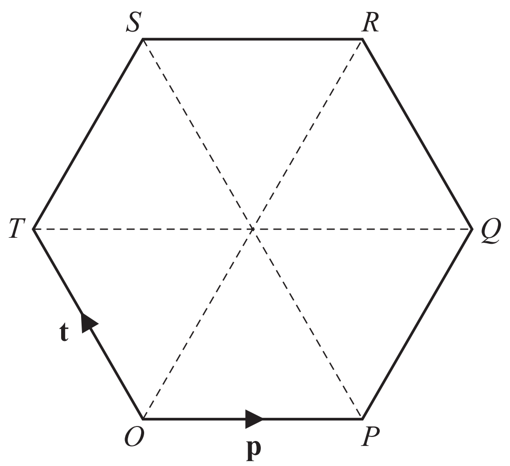
(b) $\overrightarrow{DB}$,

Answer(b) $\overrightarrow{DB}$ = ... [2]

(c) the position vector of *E*.

Answer(c) .... [2]

19



O is the origin and $OPQRST$ is a regular hexagon.

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OT} = \mathbf{t}$.

Find, in terms of $\mathbf{p}$ and $\mathbf{t}$, in their simplest forms,

(a) $\overrightarrow{PT}$,

Answer(a) $\overrightarrow{PT}$ = ... [1]

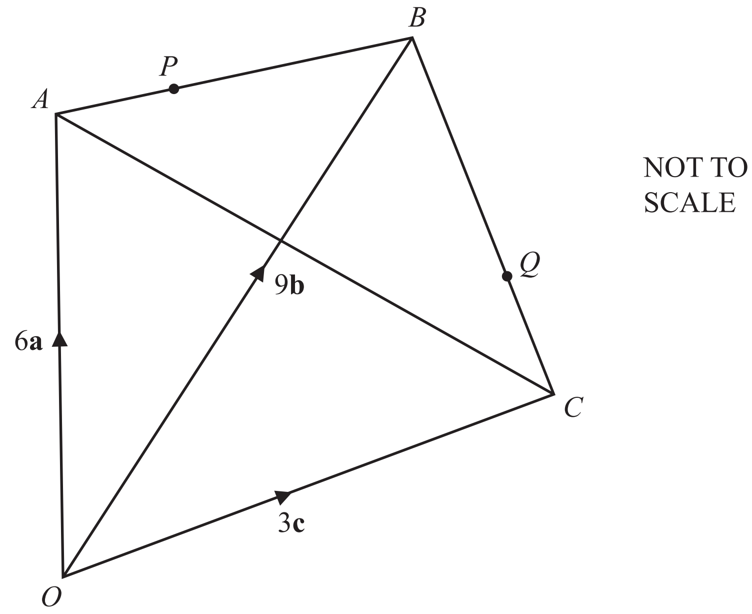
(b) $\overrightarrow{PR}$,

Answer(b) $\overrightarrow{PR}$ = ... [2]

(c) the position vector of R.

Answer(c) .... [2]

8



In the diagram, O is the origin and $\overrightarrow{OA} = 6\mathbf{a}$, $\overrightarrow{OB} = 9\mathbf{b}$ and $\overrightarrow{OC} = 3\mathbf{c}$.
The point P lies on AB such that $\overrightarrow{AP} = 3\mathbf{b} - 2\mathbf{a}$.
The point Q lies on BC such that $\overrightarrow{BQ} = 2\mathbf{c} - 6\mathbf{b}$.

(a) Find, in terms of **b** and **c**, the position vector of Q.
Give your answer in its simplest form.

Answer(a) .... [2]

(b) Find, in terms of **a** and **c**, in its simplest form

(i) $\overrightarrow{AC}$,

Answer(b)(i) $\overrightarrow{AC}$ = ... [1]

(ii) $\overrightarrow{PQ}$.

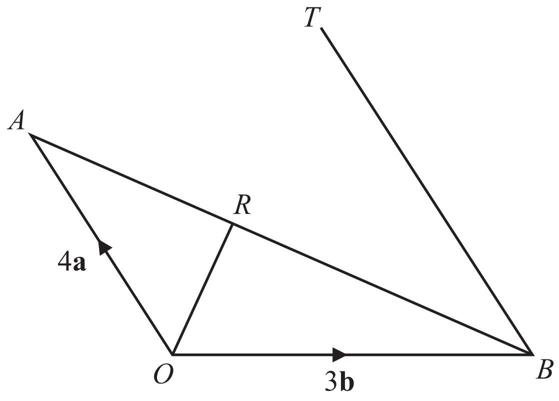
Answer(b)(ii) $\overrightarrow{PQ}$ = ... [2]

(c) Explain what your answers in **part (b)** tell you about PQ and AC.

Answer(c) ....

.... [2]

(b)



In the diagram, $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 3\mathbf{b}$.

R lies on AB such that $\overrightarrow{OR} = \frac{1}{5}(12\mathbf{a} + 6\mathbf{b})$.

T is the point such that $\overrightarrow{BT} = \frac{3}{2}\overrightarrow{OA}$.

(i) Find the following in terms of **a** and **b**, giving each answer in its simplest form.

(a) $\overrightarrow{AB}$

Answer(b)(i)(a) $\overrightarrow{AB}$ = ... [1]

(b) $\overrightarrow{AR}$

Answer(b)(i)(b) $\overrightarrow{AR}$ = ... [2]

(c) $\overrightarrow{OT}$

Answer(b)(i)(c) $\overrightarrow{OT}$ = ... [1]

(ii) Complete the following statement.

The points O, R and T are in a straight line because ....

.... [1]

(iii) Triangle OAR and triangle TBR are similar.

Find the value of $\dfrac{\text{area of triangle } TBR}{\text{area of triangle } OAR}$.

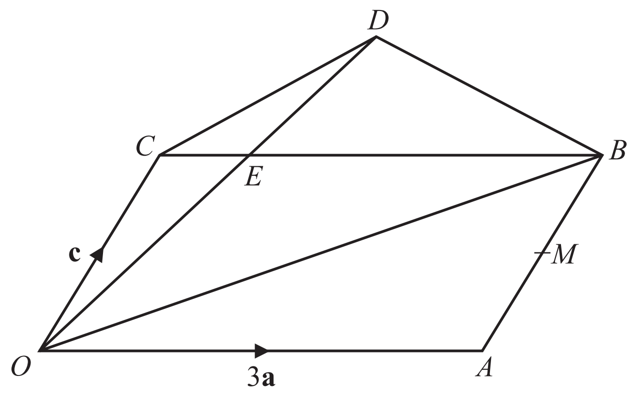
Answer(b)(iii) .... [2]

7 **(a)** P is the point (2, 5) and $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Write down the co-ordinates of Q.

Answer(a) (.......... , ...) [1]

(b)



NOT TO SCALE

O is the origin and $OABC$ is a parallelogram.
M is the midpoint of AB.

$\overrightarrow{OC} = \mathbf{c}$, $\overrightarrow{OA} = 3\mathbf{a}$ and $CE = \frac{1}{3}CB$.

OED is a straight line with $OE : ED = 2 : 1$.

Find in terms of **a** and **c**, in their simplest forms

(i) $\overrightarrow{OB}$,

Answer(b)(i) $\overrightarrow{OB}$ = ... [1]

(ii) the position vector of M,

Answer(b)(ii) .... [2]

(iii) $\overrightarrow{OE}$,

Answer(b)(iii) $\overrightarrow{OE}$ = ... [1]

(iv) $\overrightarrow{CD}$.

Answer(b)(iv) $\overrightarrow{CD}$ = ... [2]

(c) Write down two facts about the lines CD and OB.

Answer (c) ....

.... [2]