Mensuration

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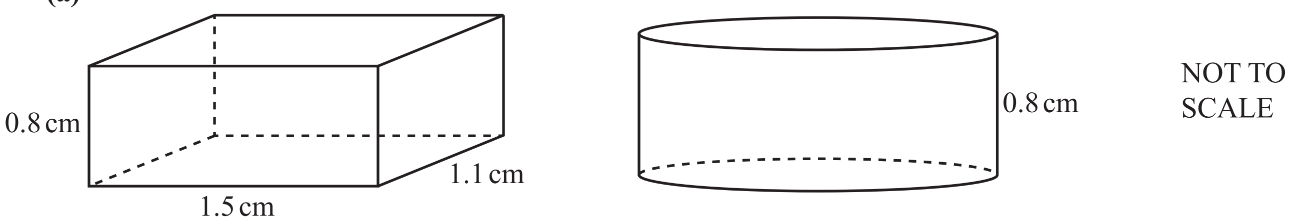
6 **(a)**



The diagram shows two sweets.
The cuboid has length 1.5 cm, width 1.1 cm and height 0.8 cm.
The cylinder has height 0.8 cm and the same volume as the cuboid.

(i) Calculate the volume of the cuboid.

.... cm^3 [2]

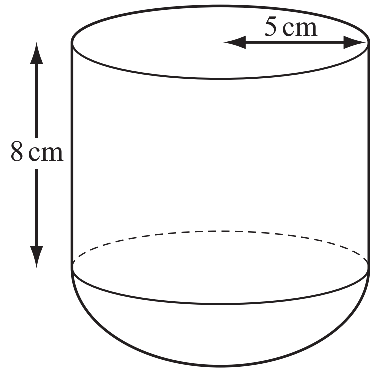
(ii) Calculate the radius of the cylinder.

.... cm [2]

(iii) Calculate the difference between the surface areas of the two sweets.

.... cm^2 [5]

17 The diagram shows a child's toy.



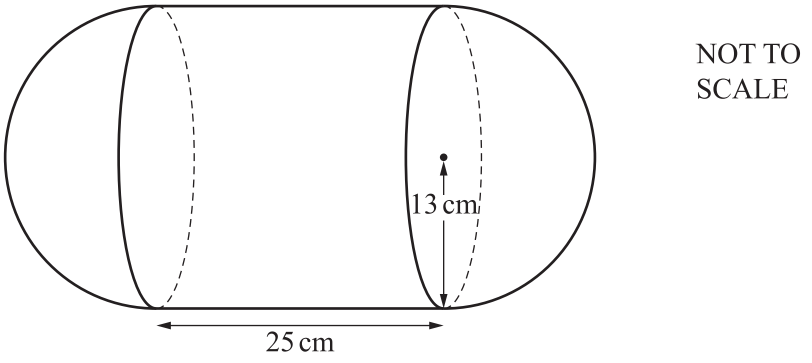
NOT TO SCALE

The shape of the toy is a cylinder of radius 5 cm and height 8 cm on top of a hemisphere of radius 5 cm.

Calculate the volume of the toy.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Answer .... cm^3 [5]

3 **(a)**



The diagram shows a solid made up of a cylinder and two hemispheres.
The radius of the cylinder and the hemispheres is 13 cm.
The length of the cylinder is 25 cm.

(i) One cubic centimetre of the solid has a mass of 2.3 g.

Calculate the mass of the solid.
Give your answer in kilograms.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

.... kg [4]

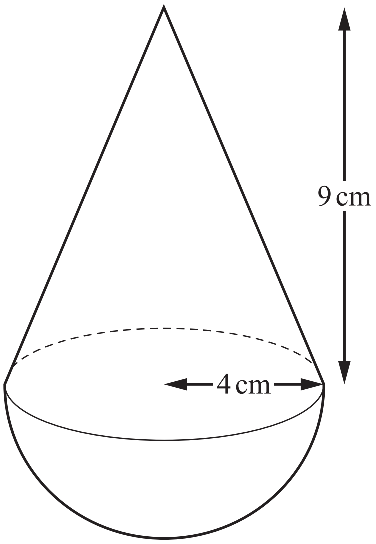
(ii) The surface of the solid is painted at a cost of \$4.70 per square metre.

Calculate the cost of painting the solid.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

\$.. [4]

21



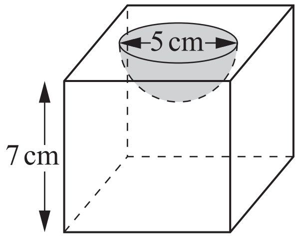
The diagram shows a toy.
The shape of the toy is a cone, with radius 4 cm and height 9 cm, on top of a hemisphere with radius 4 cm.

Calculate the volume of the toy.
Give your answer correct to the nearest cubic centimetre.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Answer .... cm^3 [4]

15 A solid consists of a metal cube with a hemisphere cut out of it.



NOT TO SCALE

The length of a side of the cube is 7 cm.
The diameter of the hemisphere is 5 cm.

Calculate the volume of this solid.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

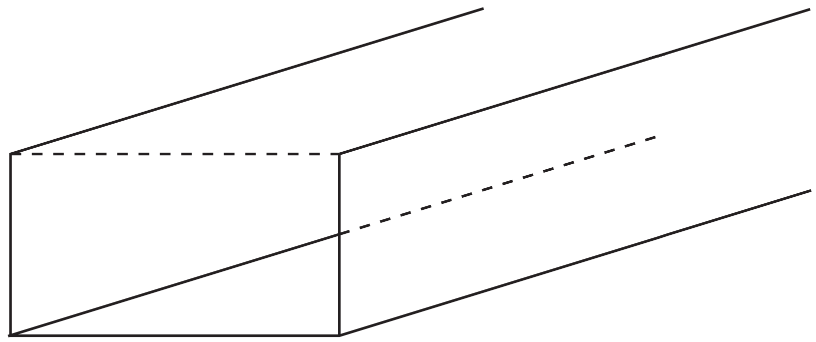
....cm^3 [3]

16 y is directly proportional to $(x + 2)^2$.
When $x = 8$, $y = 250$.

Find y when $x = 4$.

y = ... [3]

14

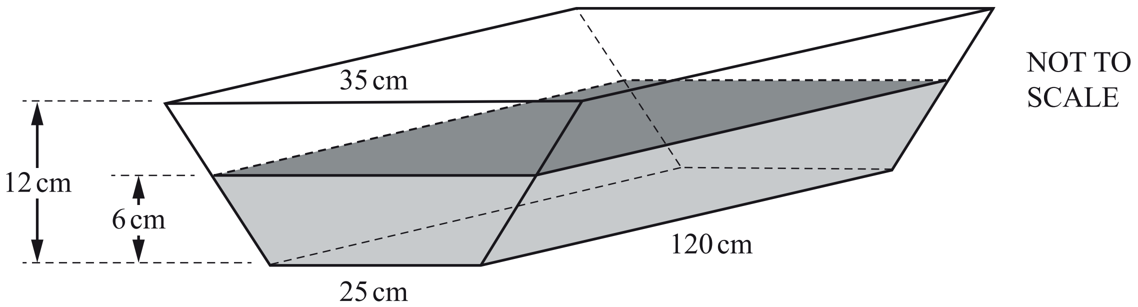


The diagram shows a channel for water.
The channel lies on horizontal ground.
This channel has a constant rectangular cross section with area 0.95 m^2.
The channel is full and the water flows through the channel at a rate of 4 metres/**minute**.

Calculate the number of cubic metres of water that flow along the channel in 3 **hours**.

Answer .... m^3 [3]

3 The diagram shows a horizontal water trough in the shape of a prism.



The cross section of this prism is a trapezium.
The trapezium has parallel sides of lengths 35 cm and 25 cm and a perpendicular height of 12 cm.
The length of the prism is 120 cm.

(a) Calculate the volume of the trough.

Answer(a) .... cm^3 [3]

(b) The trough contains water to a depth of 6 cm.

(i) Show that the volume of water is 19 800 cm^3.

Answer (b)(i)

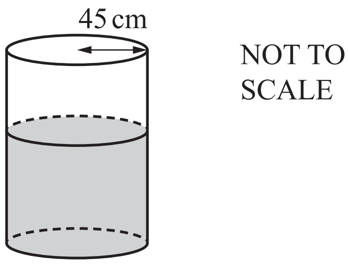
[2]

(ii) Calculate the percentage of the trough that contains water.

Answer(b)(ii) .... % [1]

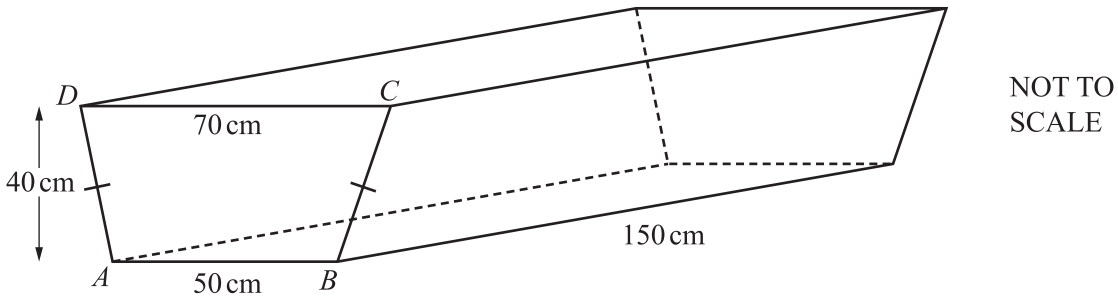
8 **(a)** A cylindrical tank contains $180\,000\,\text{cm}^3$ of water.
The radius of the tank is 45 cm.

Calculate the height of water in the tank.



Answer(a) .... cm [2]

(b)



The diagram shows an empty tank in the shape of a horizontal prism of length 150 cm.
The cross section of the prism is an isosceles trapezium *ABCD*.
AB = 50 cm, *CD* = 70 cm and the vertical height of the trapezium is 40 cm.

(i) Calculate the volume of the tank.

Answer(b)(i) .... cm^3 [3]

(ii) Write your answer to **part (b)(i)** in litres.

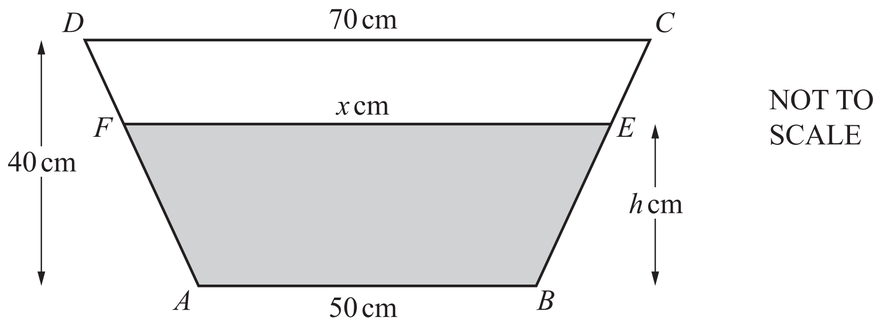
Answer(b)(ii) .... litres [1]

(c) The $180\,000\,\text{cm}^3$ of water flows from the tank in **part (a)** into the tank in **part (b)** at a rate of $15\,\text{cm}^3/\text{s}$.

Calculate the time this takes.
Give your answer in hours and minutes.

Answer(c) .... h .... min [3]

(d)



The 180 000 cm^3 of water reaches the level EF as shown above.
$EF = x$ cm and the height of the water is h cm.

(i) Using the properties of similar triangles, show that $h = 2(x - 50)$.

Answer(d)(i)

[2]

(ii) Using $h = 2(x - 50)$, show that the shaded area, in cm^2, is $x^2 - 2500$.

Answer(d)(ii)

[1]

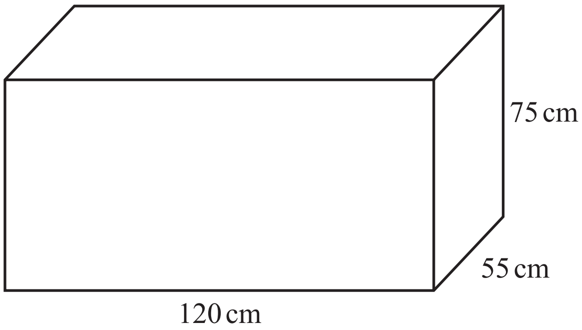
(iii) Find the value of x.

Answer(d)(iii) x = ... [2]

(iv) Find the value of h.

Answer(d)(iv) h = ... [1]

7



The diagram shows a water tank in the shape of a cuboid measuring 120 cm by 55 cm by 75 cm.
The tank is filled completely with water.

(a) Show that the capacity of the water tank is 495 litres.

Answer(a)

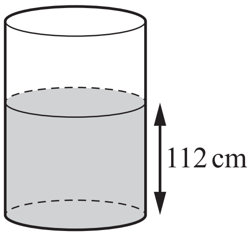
[2]

(b) **(i)** The water from the tank flows into an empty cylinder at a uniform rate of 750 millilitres per second.

Calculate the length of time, in minutes, for the water to be completely emptied from the tank.

Answer(b)(i) .... min [2]

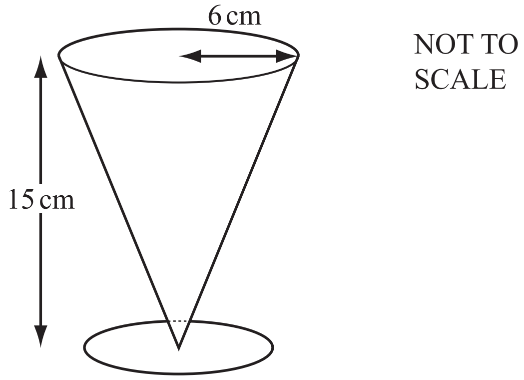
(ii) When the tank is completely empty, the height of the water in the cylinder is 112 cm.



Calculate the radius of the cylinder.

Answer(b)(ii) .... cm [3]

18



The diagram shows a glass, in the shape of a cone, for drinking milk.
The cone has a radius of 6 cm and height 15 cm.
A bottle of milk holds 2 litres.

(a) How many times can the glass be completely filled from the bottle?
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(a) .... [4]

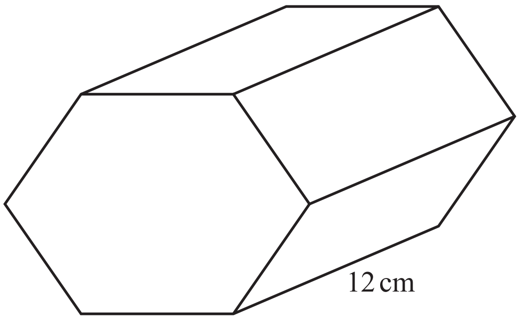
(b) Calculate the volume of milk left in the bottle.
Give your answer in cm^3.

Answer(b) .... cm^3 [3]

Question 19 is printed on the next page.

For Examiner's Use

9 (a)



The diagram shows a prism of length 12 cm.
The cross section is a regular hexagon of side 4 cm.

Calculate the total surface area of the prism.

Answer(a) .... cm^2 [4]

(b) Water flows through a cylindrical pipe of radius 0.74 cm.
It fills a 12 litre bucket in 4 minutes.

(i) Calculate the speed of the water through the pipe in centimetres per minute.

Answer(b)(i) .... cm/min [4]

(ii) When the 12 litre bucket is emptied into a circular pool, the water level rises by 5 **millimetres**.

Calculate the radius of the pool correct to the nearest centimetre.

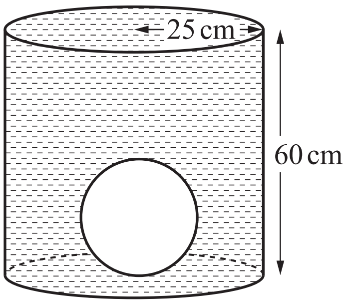
Answer(b)(ii) .... cm [5]

For Examiner's Use

4 **(a)** Calculate the volume of a metal sphere of radius 15 cm and show that it rounds to 14 140 cm^3, correct to 4 significant figures.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[2]

(b) **(i)** The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm.
The tank is filled with water.

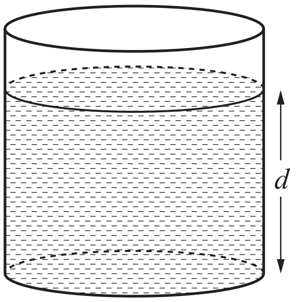


NOT TO SCALE

Calculate the volume of water required to fill the tank.

.... cm^3 [3]

(ii) The sphere is removed from the tank.



NOT TO SCALE

Calculate the depth, d, of water in the tank.

d = ... cm [2]

(c) The sphere is melted down and the metal is made into a solid cone of height 54 cm.

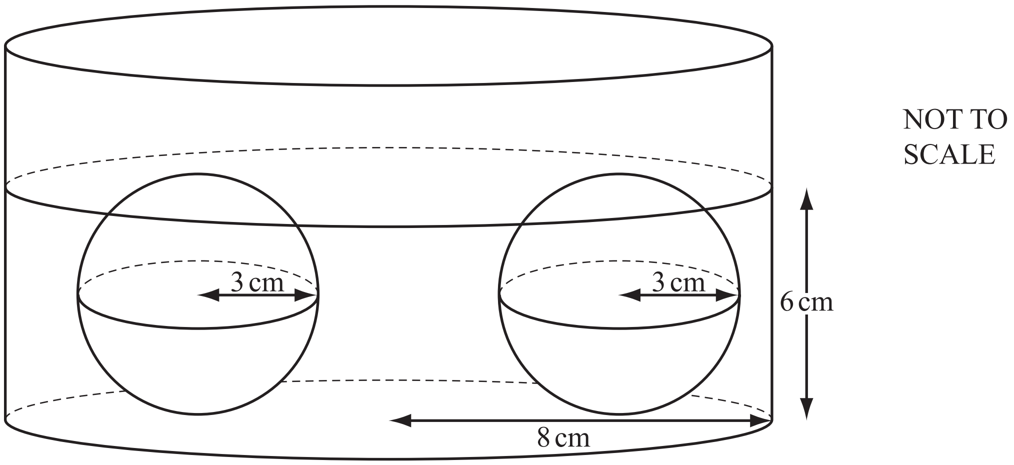
(i) Calculate the radius of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

.... cm [3]

(ii) Calculate the **total** surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

.... cm^2 [4]

5



The diagram shows two solid spheres of radius 3 cm lying on the base of a cylinder of radius 8 cm.

Liquid is poured into the cylinder until the spheres are just covered.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

(a) Calculate the volume of liquid in the cylinder in

(i) cm^3,

Answer(a)(i) .... cm^3 [4]

(ii) litres.

Answer(a)(ii) .... litres [1]

For Examiner's Use

(b) One cubic centimetre of the liquid has a mass of 1.22 grams.

Calculate the mass of the liquid in the cylinder.

Give your answer in kilograms.

Answer(b) .... kg [2]

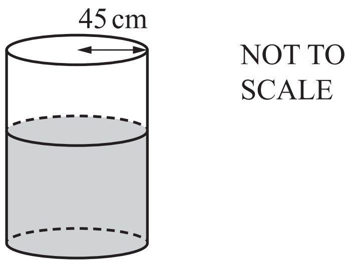
(c) The spheres are removed from the cylinder.

Calculate the new height of the liquid in the cylinder.

Answer(c) .... cm [2]

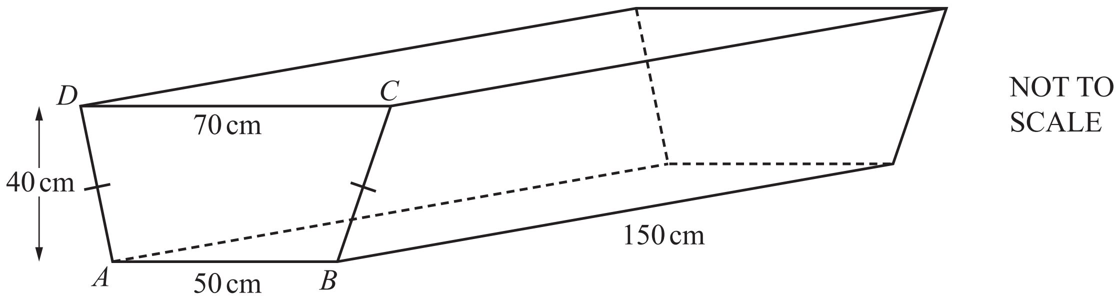
8 **(a)** A cylindrical tank contains 180 000 cm^3 of water.
The radius of the tank is 45 cm.

Calculate the height of water in the tank.



Answer(a) .... cm [2]

(b)



The diagram shows an empty tank in the shape of a horizontal prism of length 150 cm.
The cross section of the prism is an isosceles trapezium *ABCD*.
AB = 50 cm, *CD* = 70 cm and the vertical height of the trapezium is 40 cm.

(i) Calculate the volume of the tank.

Answer(b)(i) .... cm^3 [3]

(ii) Write your answer to **part (b)(i)** in litres.

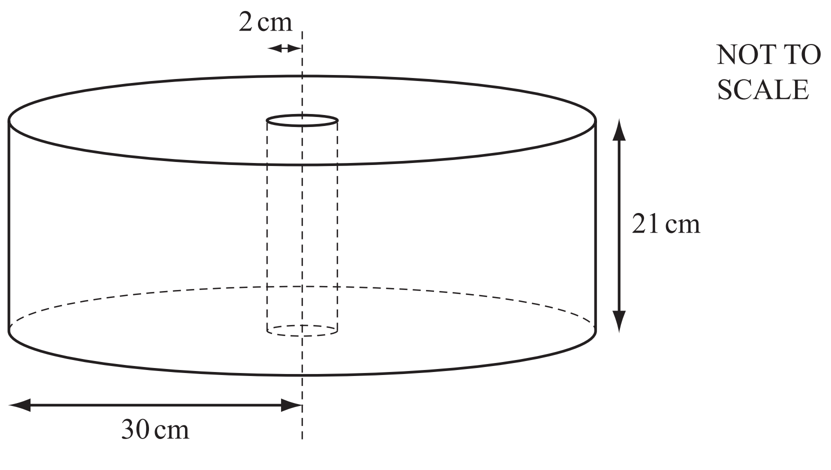
Answer(b)(ii) .... litres [1]

(c) The 180 000 cm^3 of water flows from the tank in **part (a)** into the tank in **part (b)** at a rate of 15 cm^3/s.

Calculate the time this takes.
Give your answer in hours and minutes.

Answer(c) .... h .... min [3]

(d)

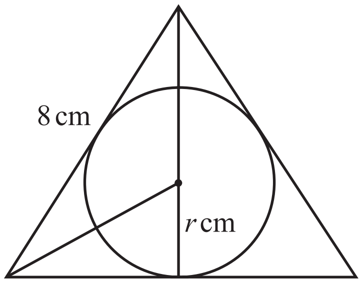


Paper is sold in cylindrical rolls.
There is a wooden cylinder of radius 2 cm and height 21 cm in the centre of each roll.
The outer radius of a roll of paper is 30 cm.

(i) Calculate the volume of paper in a roll.

Answer(d)(i) .... cm^3 [3]

10 (a)



NOT TO SCALE

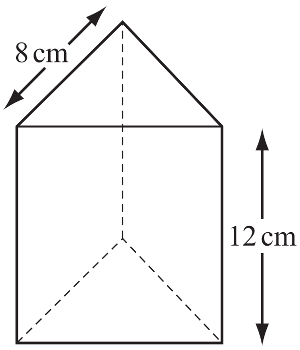
The three sides of an equilateral triangle are tangents to a circle of radius r cm.
The sides of the triangle are 8 cm long.

Calculate the value of r.
Show that it rounds to 2.3, correct to 1 decimal place.

Answer(a)

[3]

(b)



NOT TO SCALE

The diagram shows a box in the shape of a triangular prism of height 12 cm.
The cross section is an equilateral triangle of side 8 cm.

Calculate the volume of the box.

Answer(b) .... cm^3 [4]

(c) The box contains biscuits.
Each biscuit is a cylinder of radius 2.3 centimetres and height 4 millimetres.

Calculate

(i) the largest number of biscuits that can be placed in the box,

Answer(c)(i) .... [3]

(ii) the volume of one biscuit in cubic centimetres,

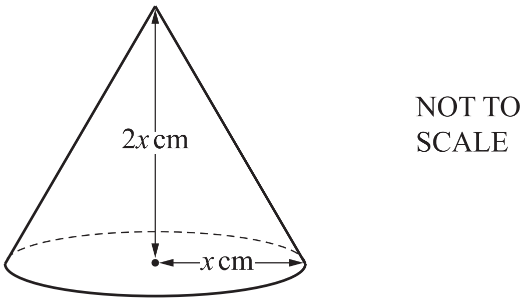
Answer(c)(ii) .... cm^3 [2]

(iii) the percentage of the volume of the box **not** filled with biscuits.

Answer(c)(iii) .... % [3]

Question 11 is printed on the next page.

(b)



The cone in the diagram has radius x cm and height $2x$ cm.
The volume of the cone is $500\,\text{cm}^3$.

Find the value of x.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

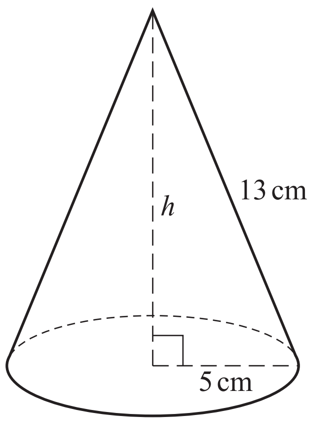
x = ... [3]

(c) Two mathematically similar solids have volumes of $180\,\text{cm}^3$ and $360\,\text{cm}^3$.
The surface area of the smaller solid is $180\,\text{cm}^2$.

Calculate the surface area of the larger solid.

.... cm^2 [3]

3



NOT TO SCALE

(a) The diagram shows a cone of radius 5 cm and slant height 13 cm.

(i) Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

Answer(a)(i) .... cm^2 [2]

(ii) Calculate the perpendicular height, h, of the cone.

Answer(a)(ii) h = ... cm [3]

(iii) Calculate the volume of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(a)(iii) .... cm^3 [2]

(iv) Write your answer to **part (a)(iii)** in cubic metres.
Give your answer in standard form.

Answer(a)(iv) .... m^3 [2]

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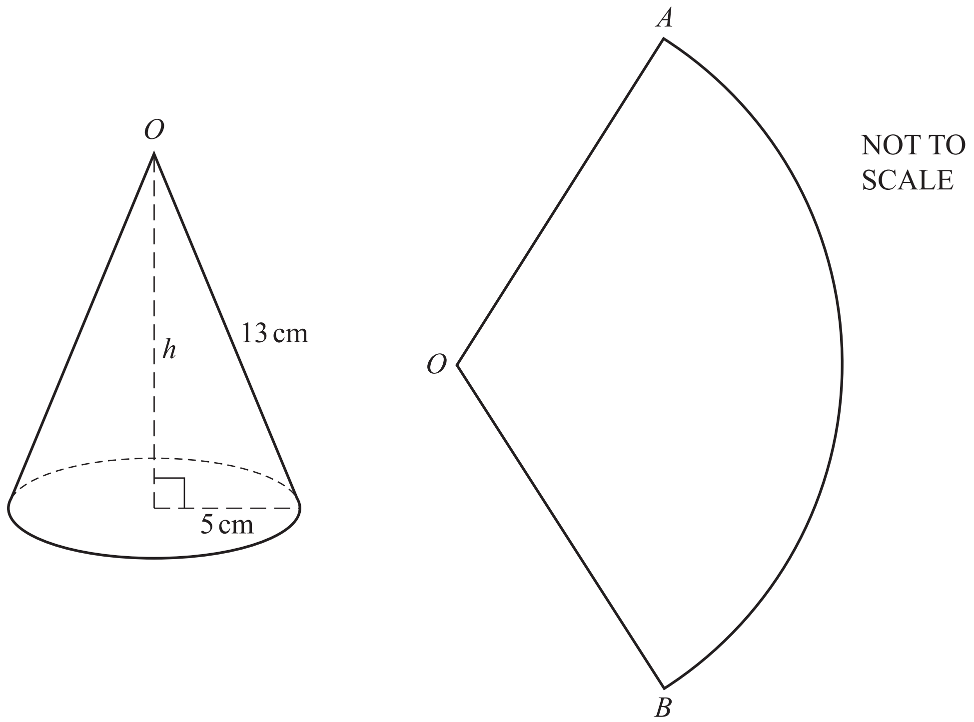
(b)

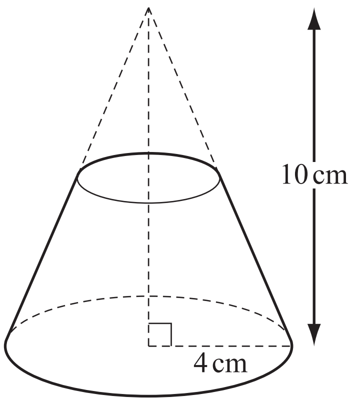


The cone is now cut along a slant height and it opens out to make the sector *AOB* of a circle.

Calculate angle *AOB*.

Answer(b) Angle *AOB* = ... [4]

18



A **solid** cone has base radius 4 cm and height 10 cm.
A mathematically similar cone is removed from the top as shown in the diagram.
The volume of the cone that is removed is $\frac{1}{8}$ of the volume of the original cone.

(a) Explain why the cone that is removed has radius 2 cm and height 5 cm.

Answer(a)

[2]

(b) Calculate the volume of the remaining solid.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(b) .... cm^3 [4]

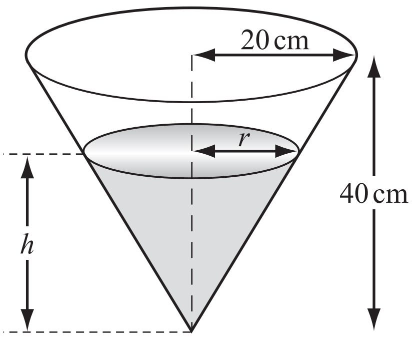
Question 19 is printed on the next page.

(c) The container has a height of 35 cm.

Calculate the capacity of the container.
Give your answer in litres.

Answer(c) .... litres [3]

(d) Sandra's container is completely filled with water.
All the water is then poured into another container in the shape of a cone.
The cone has radius 20 cm and height 40 cm.



(i) The diagram shows the water in the cone.

Show that $r = \frac{h}{2}$.

Answer(d)(i)

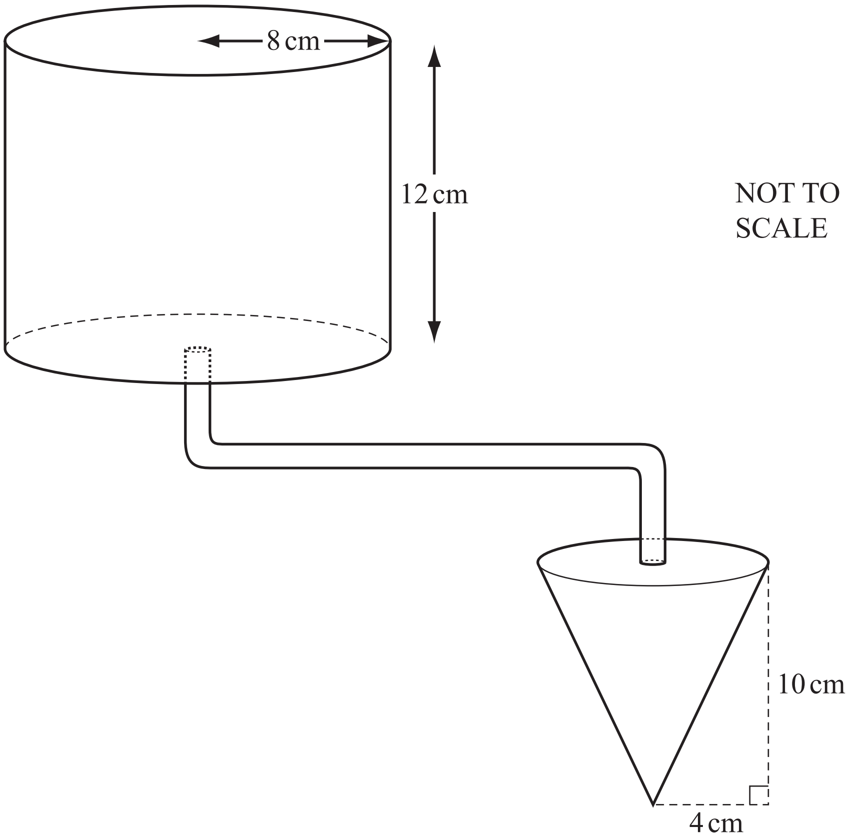
[1]

(ii) Find the height, h, of the water in the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(d)(ii) h = ... cm [3]

For Examiner's Use

5



The diagram shows a cylinder with radius 8 cm and height 12 cm which is full of water.
A pipe connects the cylinder to a cone.
The cone has radius 4 cm and height 10 cm.

(a) **(i)** Calculate the volume of water in the cylinder.
Show that it rounds to 2410 cm^3 correct to 3 significant figures.

Answer(a)(i)

[2]

(ii) Change 2410 cm^3 into litres.

Answer(a)(ii) .... litres [1]

(b) Water flows from the cylinder along the pipe into the cone at a rate of $2\,cm^3$ per second.

Calculate the time taken to fill the empty cone.
Give your answer in minutes and seconds correct to the nearest second.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

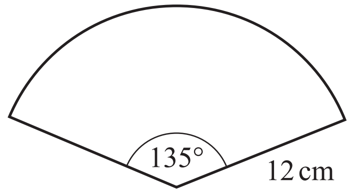
Answer(b) .... min .... s [4]

(c) Find the number of empty cones which can be filled completely from the full cylinder.

Answer(c) .... [3]

4 **(a)** A sector of a circle has radius 12 cm and an angle of 135°.

(i) Calculate the length of the arc of this sector.
Give your answer as a multiple of π.

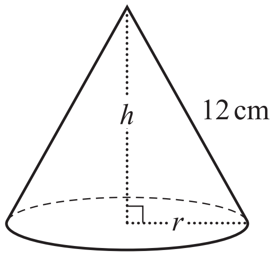


NOT TO SCALE

Answer(a)(i) .... cm [2]

(ii) The sector is used to make a cone.

(a) Calculate the base radius, *r*.



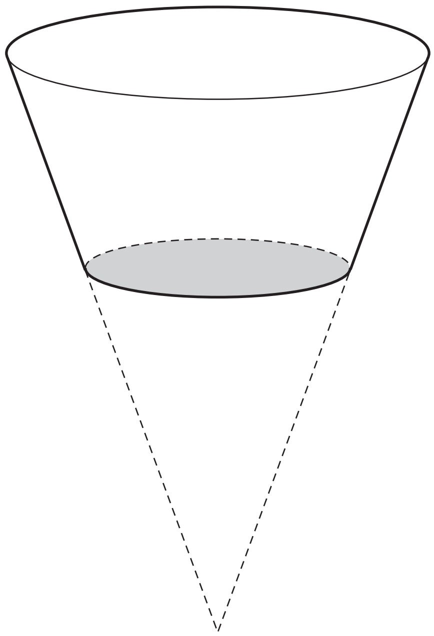
NOT TO SCALE

Answer(a)(ii)(a) *r* = ... cm [2]

(b) Calculate the height of the cone, *h*.

Answer(a)(ii)(b) *h* = ... cm [3]

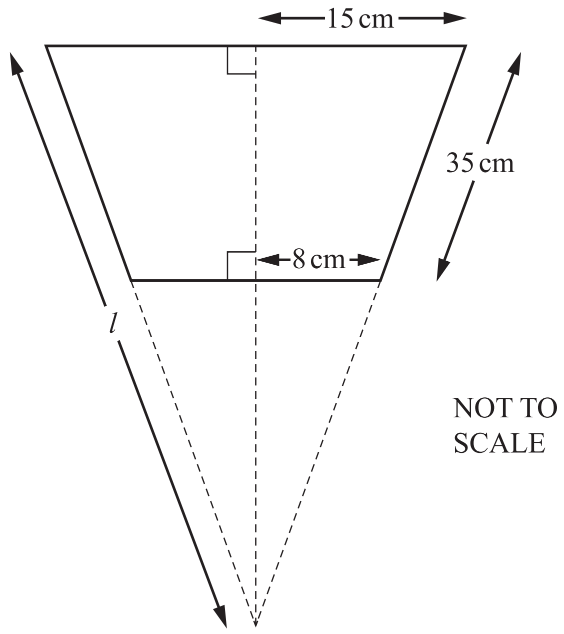
(b) The diagram shows a plant pot.
It is made by removing a small cone from a larger cone and adding a circular base.



NOT TO SCALE

This is the cross section of the plant pot.

(i) Find l.



Answer(b)(i) l = ... cm [3]

(ii) Calculate the total surface area of the outside of the plant pot.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

Answer(b)(ii) .... cm^2 [3]

(c) Some cones are mathematically similar.
For these cones, the mass, M grams, is proportional to the cube of the base radius, r cm.
One of the cones has mass 1458 grams and base radius 4.5 cm.

(i) Find an expression for M in terms of r.

Answer(c)(i) M = ... [2]

(ii) Two of the cones have radii in the ratio 2 : 3.

Write down the ratio of their masses.

Answer(c)(ii) .... : .... [1]

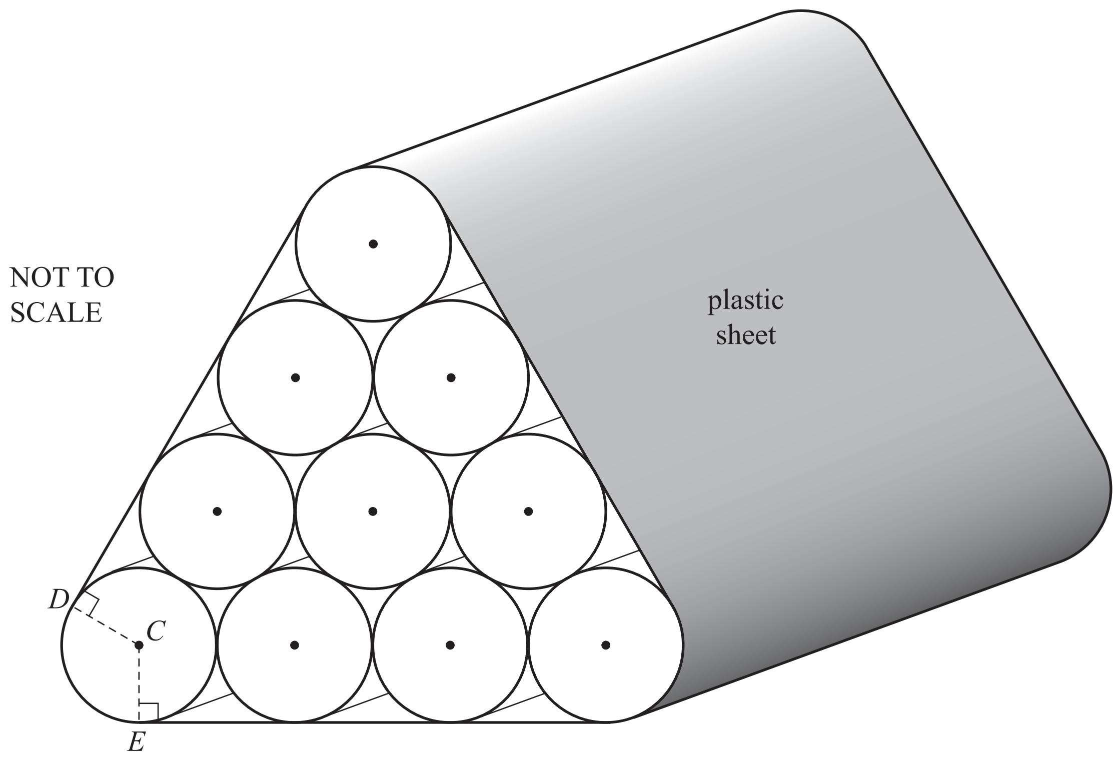
7 **(a)** Calculate the volume of a cylinder of radius 31 **centimetres** and length 15 **metres**.
Give your answer in cubic metres.

Answer(a) .... m^3 [3]

(b) A tree trunk has a circular cross-section of radius 31 cm and length 15 m.
One cubic metre of the wood has a mass of 800 kg.
Calculate the mass of the tree trunk, giving your answer in tonnes.

Answer(b) .... tonnes [2]

(c)



The diagram shows a pile of 10 tree trunks.
Each tree trunk has a circular cross-section of radius 31 cm and length 15 m.
A plastic sheet is wrapped around the pile.

C is the centre of one of the circles.
CE and *CD* are perpendicular to the straight edges, as shown.

For Examiner's Use

(i) Show that angle $ECD = 120°$.

Answer(c)(i)

[2]

(ii) Calculate the length of the arc DE, giving your answer in metres.

Answer(c)(ii) .... m [2]

(iii) The edge of the plastic sheet forms the perimeter of the cross-section of the pile.
The perimeter consists of three straight lines and three arcs.
Calculate this perimeter, giving your answer in metres.

Answer(c)(iii) .... m [3]

(iv) The plastic sheet does not cover the two ends of the pile.
Calculate the area of the plastic sheet.

Answer(c)(iv) .... m^2 [1]