X & Y graphs

www.Q8maths.com

2

$$f(x) = 3 - x - x^2 \qquad g(x) = 3^x$$

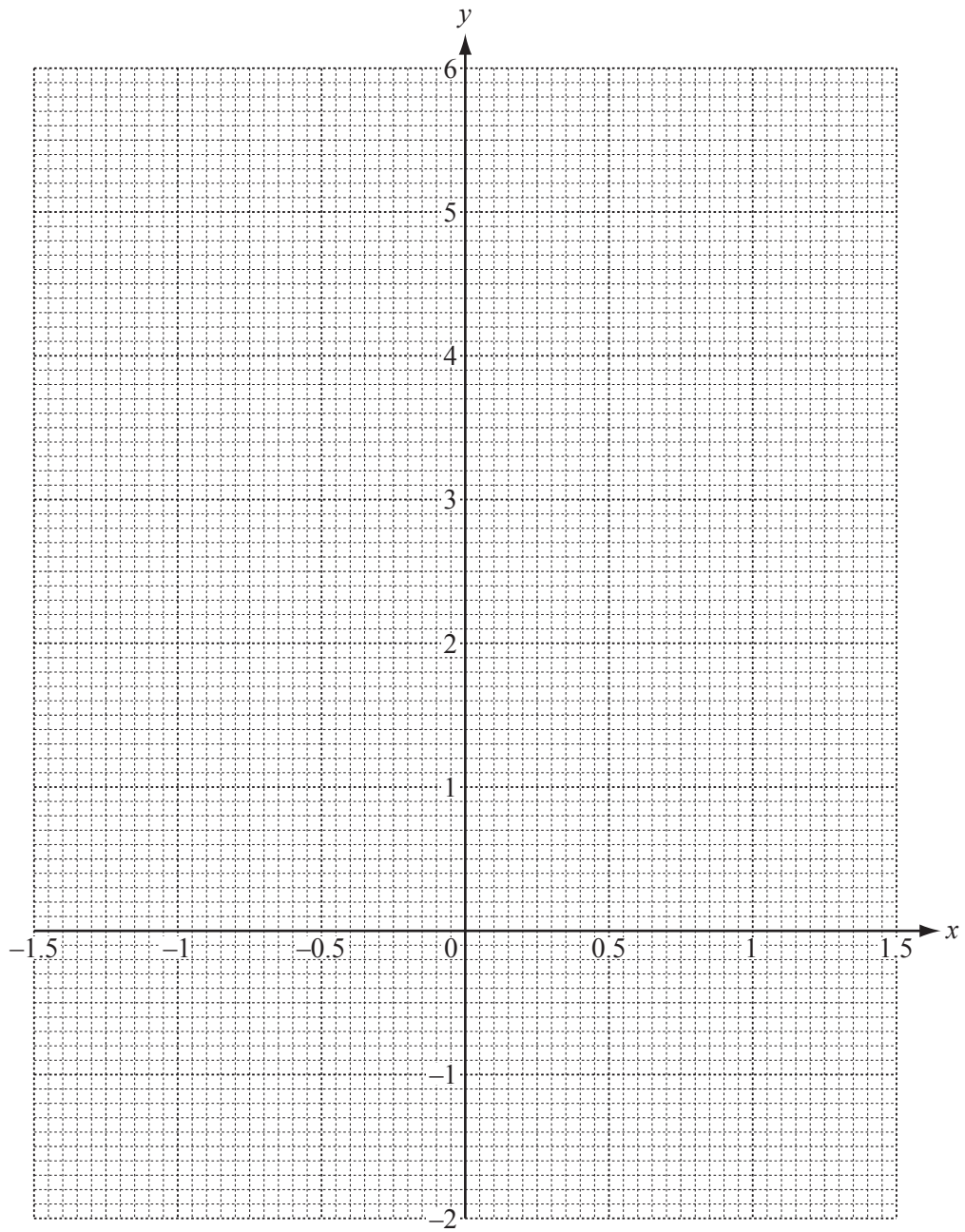
(a) Complete the tables of values for f(x) and g(x).

x	–1.5	–1	–0.5	0	0.5	1	1.5
f(x)	2.25	3	3.25		2.25	1	–0.75

x	–1.5	–1	–0.5	0	0.5	1	1.5
g(x)	0.19		0.58		1.73	3	5.20

[3]

(b) On the grid, draw the graphs of $y = f(x)$ and $y = g(x)$ for $-1.5 \leqslant x \leqslant 1.5$.



[6]

For Examiner's Use

(c) For $-1.5 \leqslant x \leqslant 1.5$, use your graphs to solve

(i) $f(x) = 0$,

Answer(c)(i) $x =$ ..... [1]

(ii) $g(x) = 4$,

Answer(c)(ii) $x =$ ..... [1]

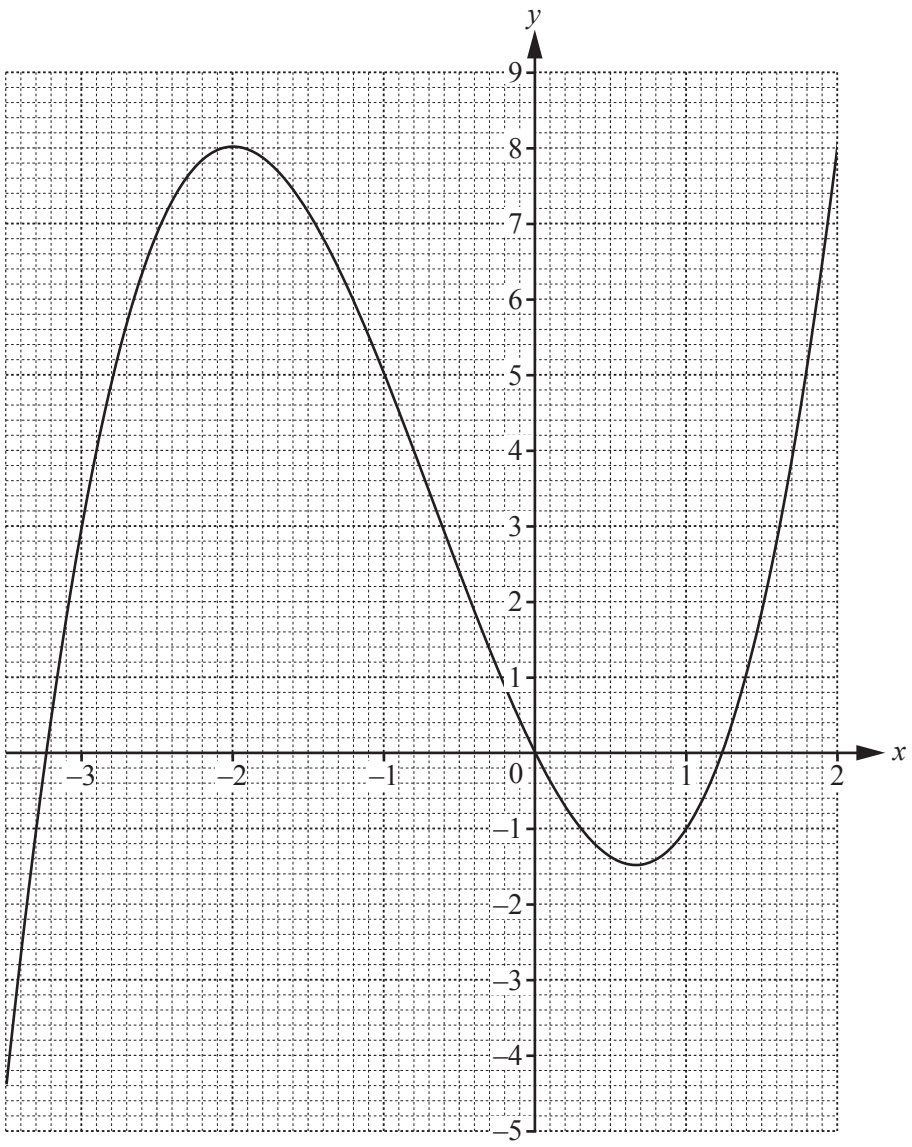
(iii) $f(x) = g(x)$.

Answer(c)(iii) $x =$ ..... [1]

(d) By drawing a suitable tangent, find an estimate of the gradient of the graph of $y = f(x)$ when $x = 0.5$.

Answer(d) ..... [3]

19 The curve $y = x^3 + 2x^2 - 4x$ is shown on the grid.



(a) By drawing a suitable tangent, find an estimate of the gradient of the curve when $x = 1$.

..... [3]

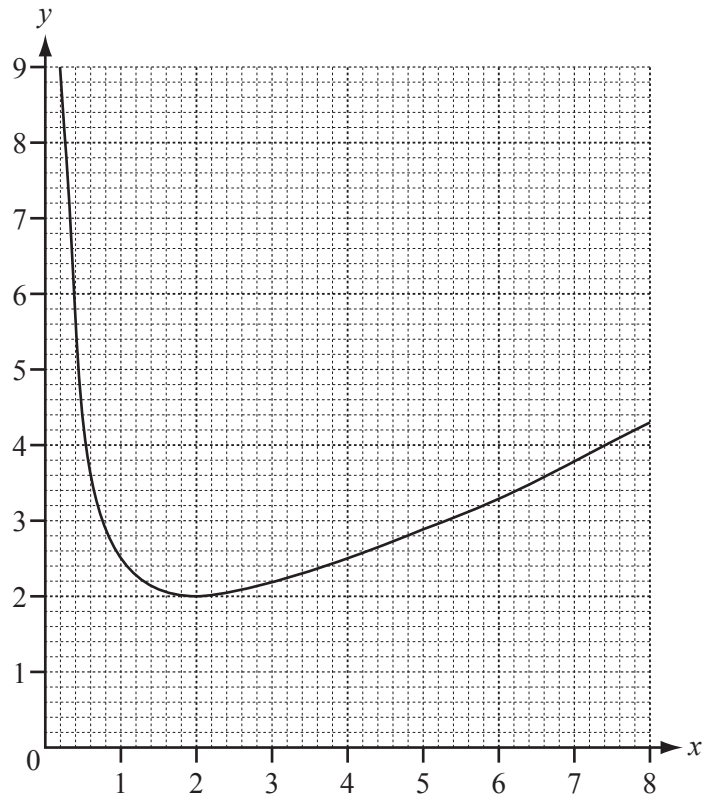
(b) A point D lies on the curve.
The x co-ordinate of D is negative.
The gradient of the tangent at D is 0.

Write down the co-ordinates of D.

(..... , ...) [1]

16

For Examiner's Use



The diagram shows the graph of $y = \frac{x}{2} + \frac{2}{x}$, for $0 < x \leqslant 8$.

(a) Use the graph to solve the equation $\frac{x}{2} + \frac{2}{x} = 3$.

Answer (a) $x =$ ..... or $x =$ ..... [2]

(b) By drawing a suitable tangent, work out an estimate of the gradient of the graph where $x = 1$.

Answer(b) ..... [3]

[Turn over

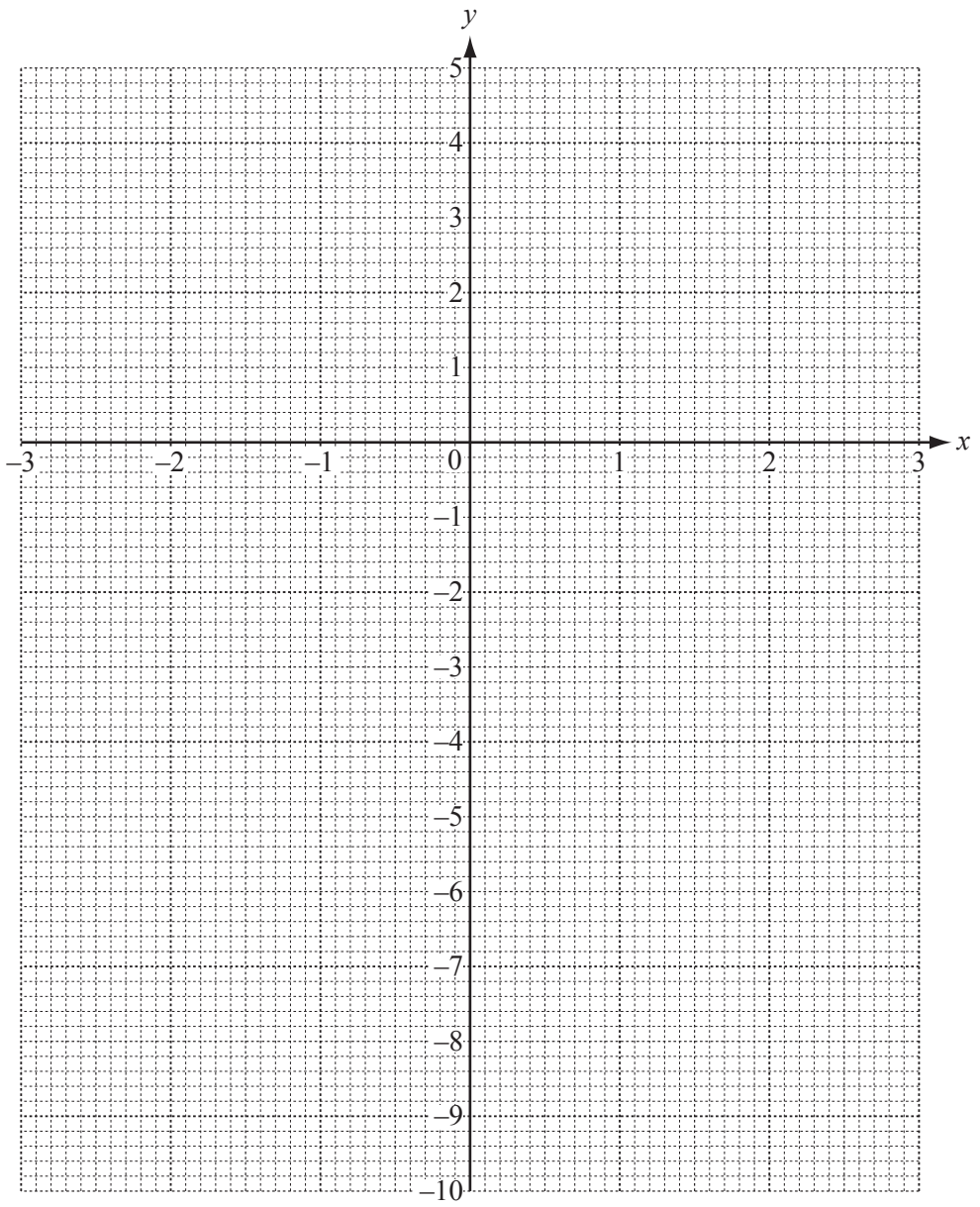
For Examiner's Use

5 **(a)** Complete this table of values for the function $f(x) = \frac{1}{x} - x^2,\ x \neq 0$.

x	–3	–2	–1	–0.5	–0.2		0.2	0.5	1	2	3
f(x)	–9.33	–4.5	–2	–2.25			4.96			–3.5	–8.67

[3]

(b) Draw the graph of $f(x) = \frac{1}{x} - x^2$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.



[5]

For Examiner's Use

(c) Use your graph to solve $f(x) = -3$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) By drawing a suitable line on your graph, solve the equation $f(x) = 2x - 2$.

Answer(d) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(e) By drawing a suitable tangent, work out an estimate of the gradient of the curve at the point where $x = -2$.

You must show your working.

Answer(e) ..... [3]

7 $f(x) = 2^x$

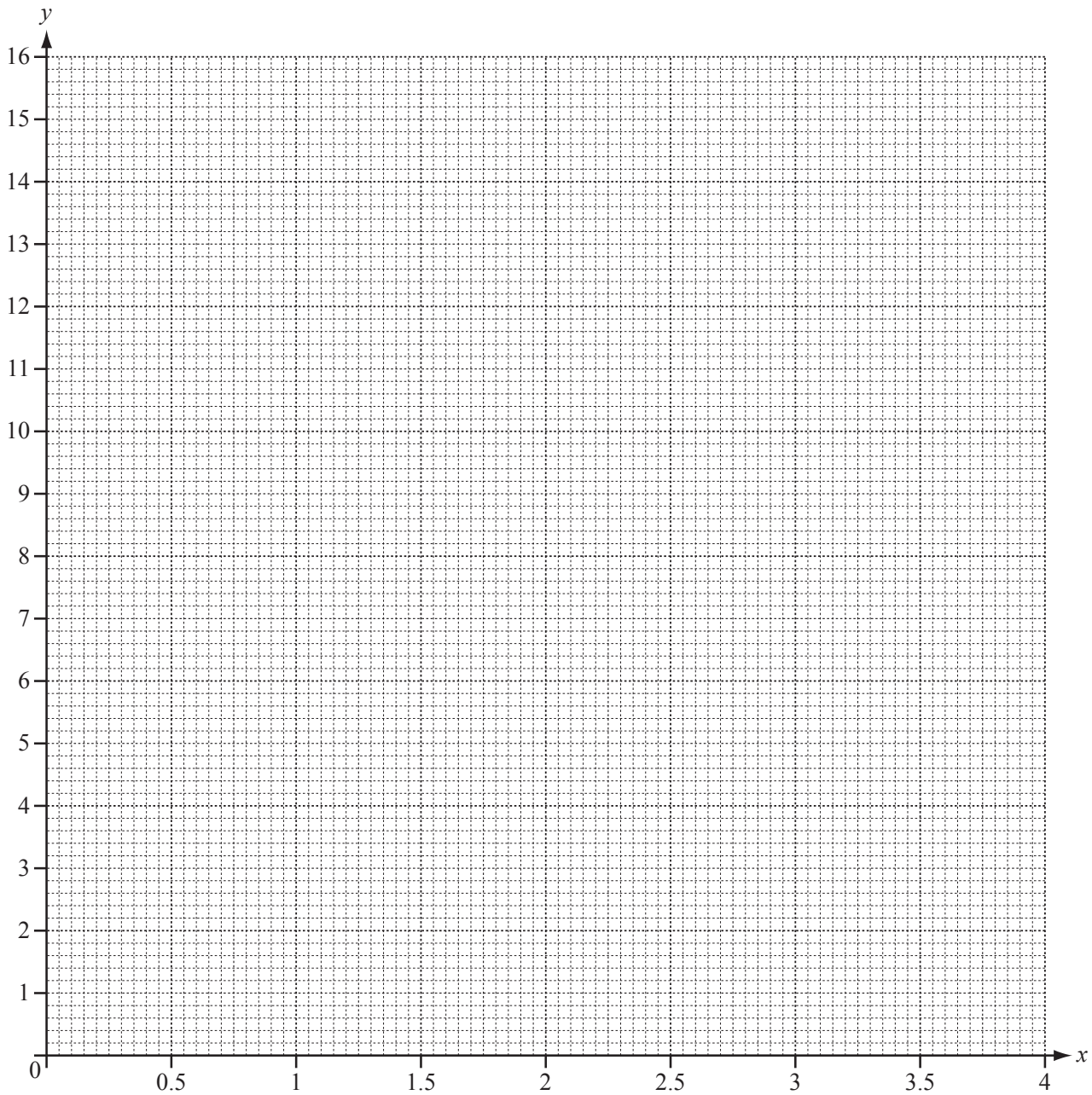
For Examiner's Use

(a) Complete the table.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$		1.4	2	2.8	4	5.7	8		

[3]

(b) Draw the graph of $y = f(x)$ for $0 \leqslant x \leqslant 4$.



[4]

(c) Use your graph to solve the equation $2^x = 5$.

For Examiner's Use

Answer(c) $x =$ ..... [1]

(d) Draw a suitable straight line and use it to solve the equation $2^x = 3x$.

Answer(d) $x =$ ..... or $x =$ ..... [3]

(e) Draw a suitable tangent and use it to find the co-ordinates of the point on the graph of $y = f(x)$ where the gradient of the graph is 3.

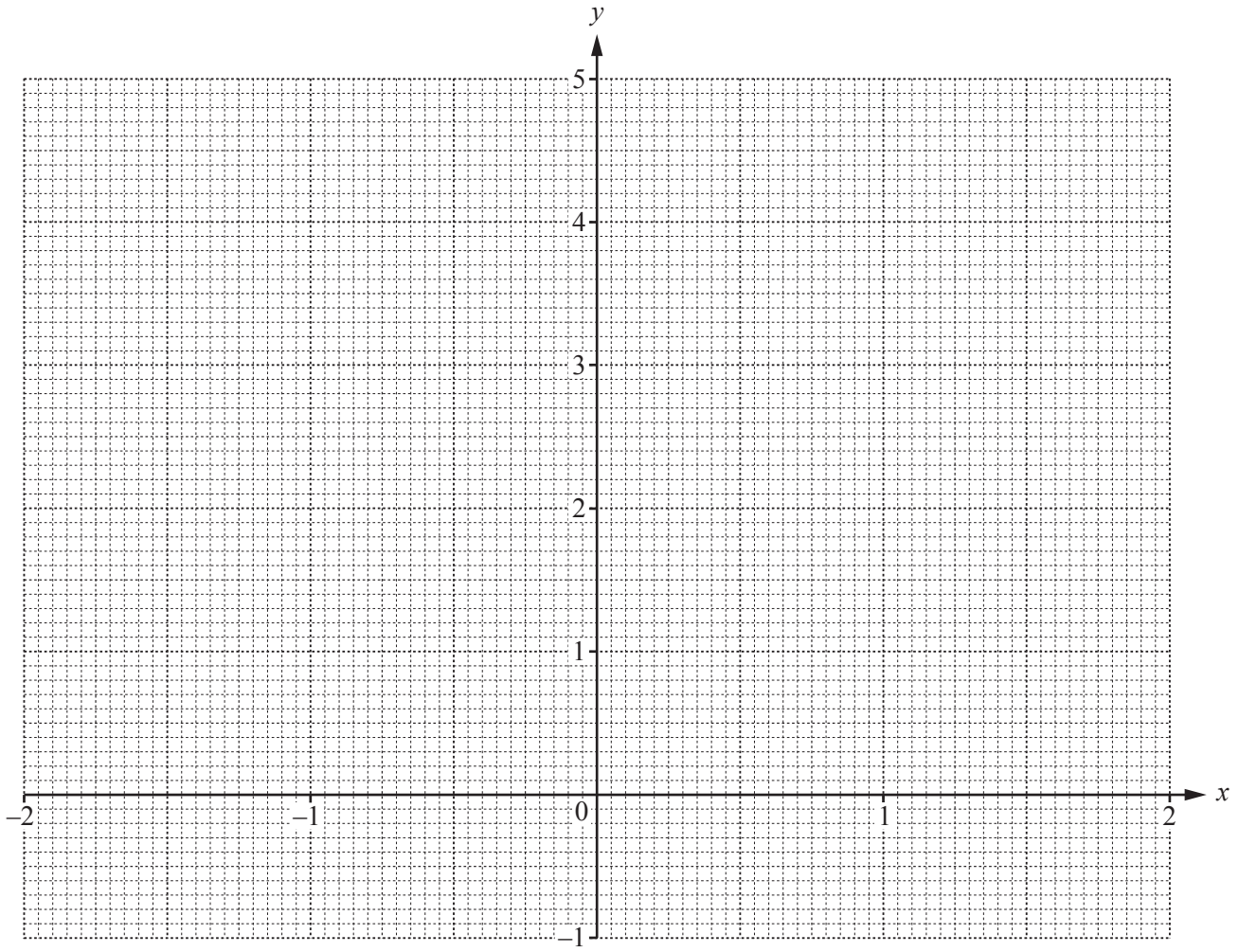
Answer(e) (..... , ...) [3]

2 The table shows some values for $y = x^3 - 3x + 2$.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y		3.125		3.375	2		0		4

(a) Complete the table of values. [4]

(b) On the grid, draw the graph of $y = x^3 - 3x + 2$ for $-2 \leqslant x \leqslant 2$.



[4]

(c) By drawing a suitable line, solve the equation $x^3 - 3x + 2 = x + 1$ for $-2 \leqslant x \leqslant 2$.

Answer(c) $x =$ ..... or $x =$ ..... [3]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.5$.

Answer(d) ..... [3]

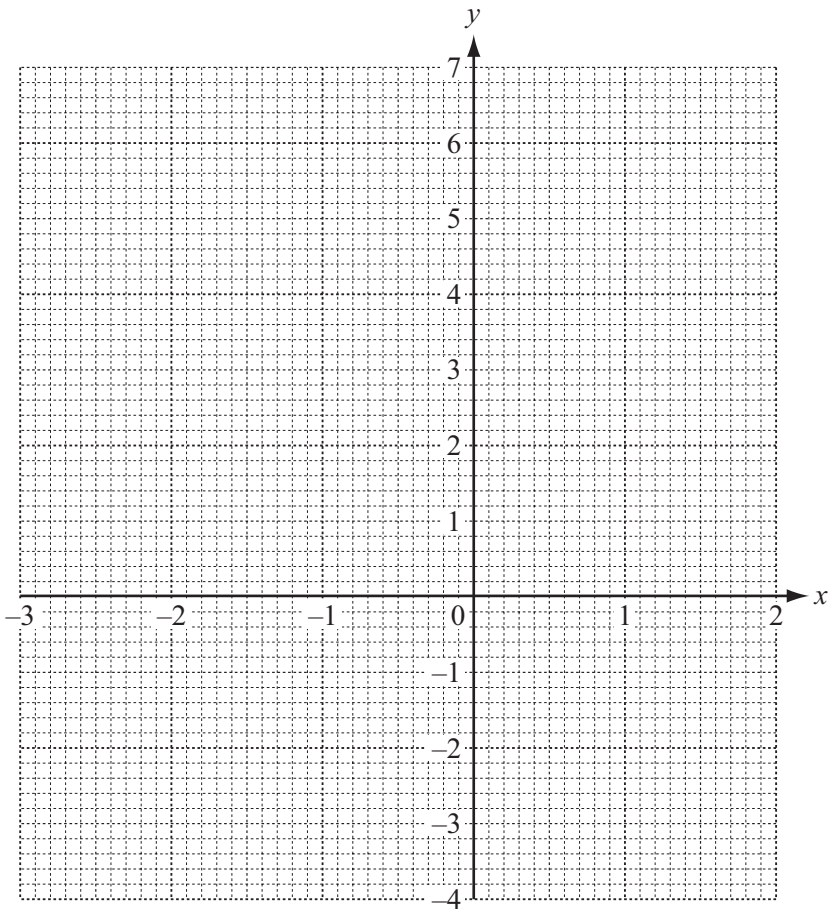
2 $f(x) = \frac{1}{x^2} - 2x$, $x \neq 0$

(a) Complete the table of values for $f(x)$.

x	–3	–2.5	–2	–1.5	–1	–0.5	0.4	0.5	1	1.5	2
$f(x)$	6.1	5.2	4.3	3.4		5	5.5			–2.6	–3.8

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 2$.



[5]

(c) Solve the equation $f(x) = 2$.

Answer(c) $x =$ ..... [1]

(d) Solve the equation $f(x) = 2x + 3$.

Answer(d) $x =$ ..... [3]

(e) **(i)** Draw the tangent to the graph of $y = f(x)$ at the point where $x = -1.5$. [1]

(ii) Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

Answer(e)(ii) ..... [2]

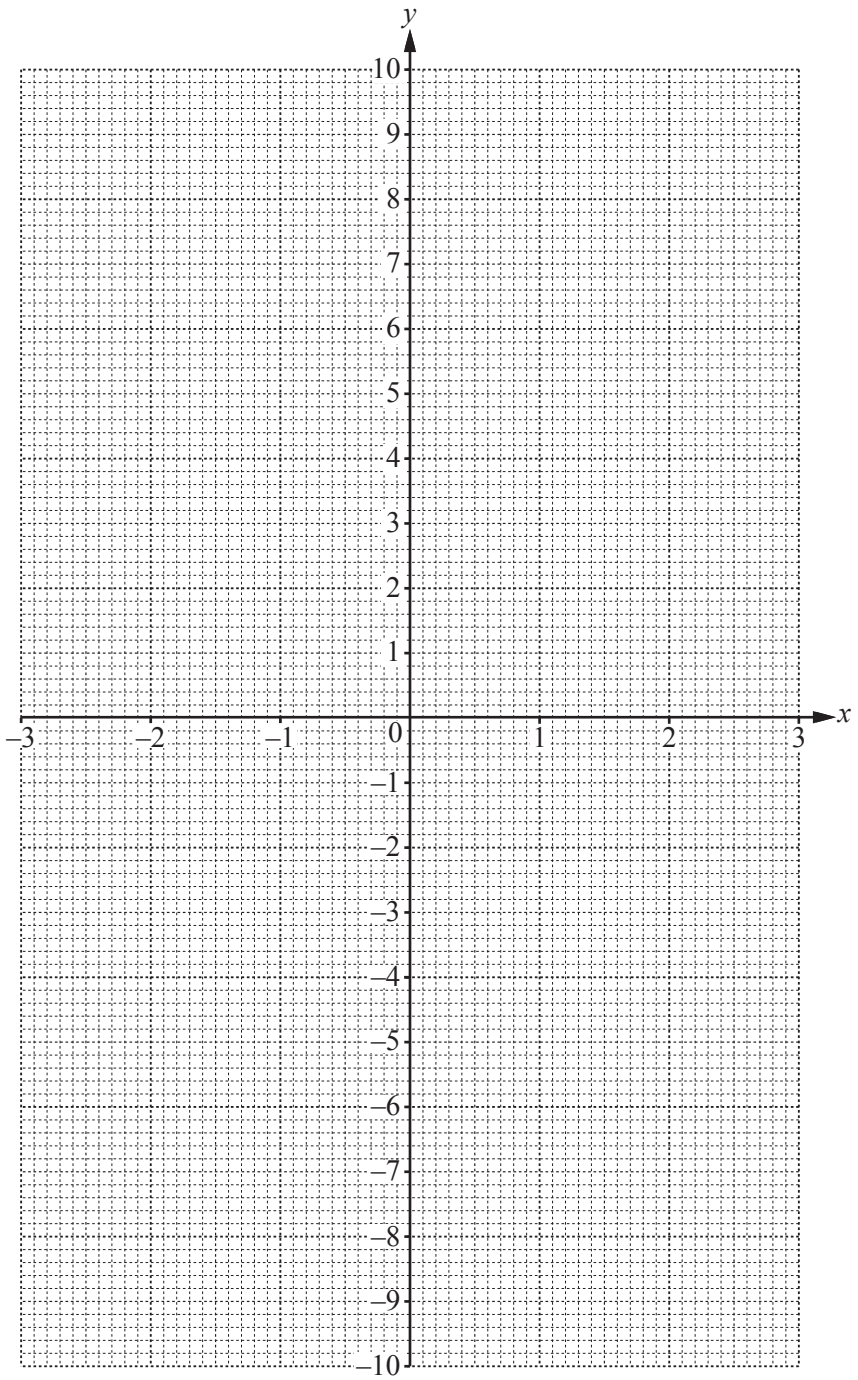
4 $f(x) = x^2 - \frac{1}{x} - 4\,,\ x \neq 0$

(a) (i) Complete the table.

x	–3	–2	–1	–0.5	–0.1		0.2	0.5	1	2	3
$f(x)$	5.3	0.5		–1.8	6.0		–9.0	–5.8	–4		4.7

[2]

(ii) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.1$ and $0.2 \leqslant x \leqslant 3$.



[5]

(b) Use your graph to solve the equation $f(x) = 0$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(c) Find an integer k, for which $f(x) = k$ has one solution.

$k =$ ..... [1]

(d) **(i)** By drawing a suitable straight line, solve the equation $f(x) + 2 = -5x$.

$x =$ ..... or $x =$ ..... [4]

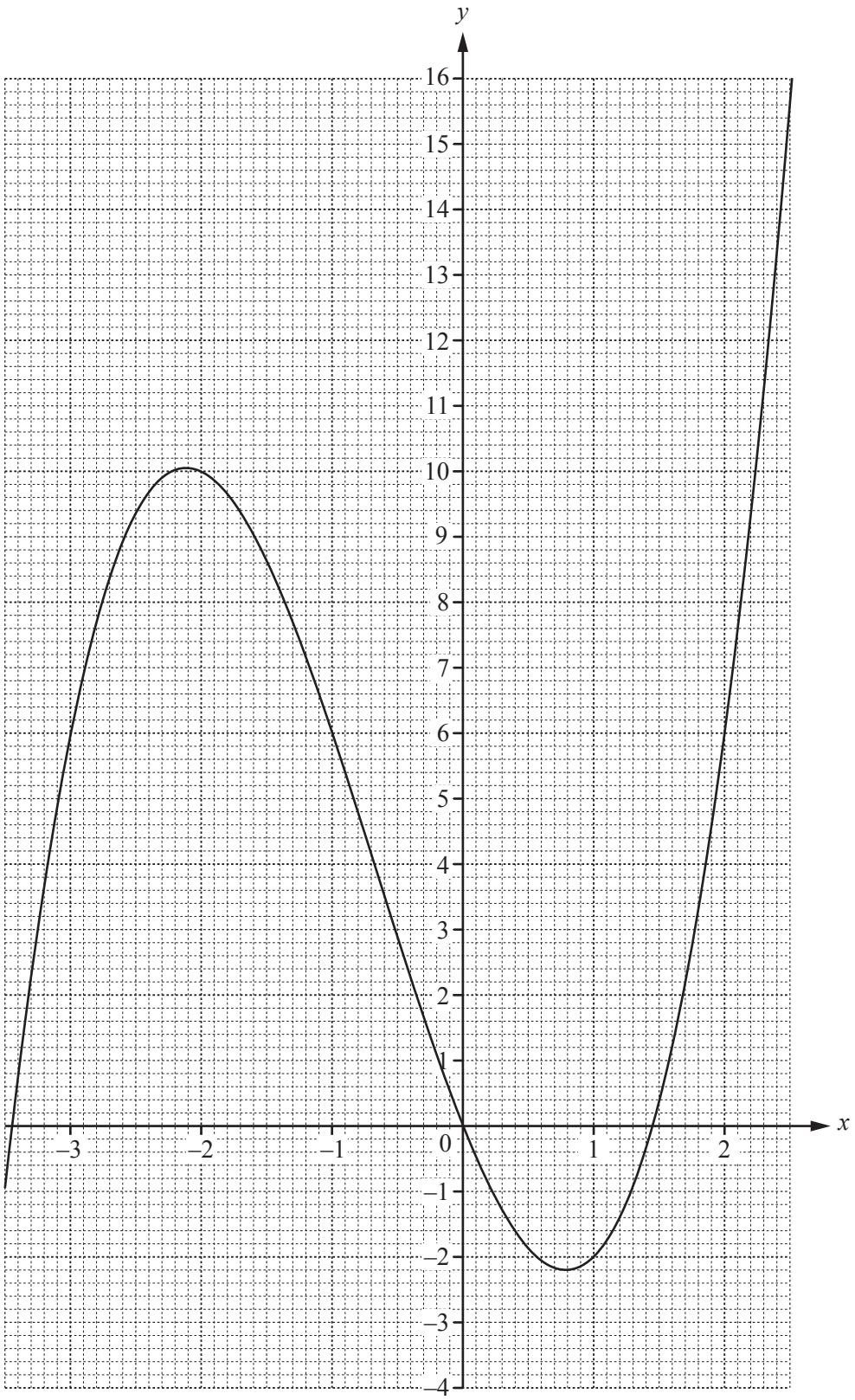
(ii) $f(x) + 2 = -5x$ can be written as $x^3 + ax^2 + bx - 1 = 0$.

Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [2]

3 The diagram shows the graph of $y = f(x)$ for $-3.5 \leqslant x \leqslant 2.5$.



(a) **(i)** Find f(– 2).

..... [1]

(ii) Solve the equation f(x) = 2.

x = ... or x = ... or x = ...[3]

(iii) Two tangents, each with gradient 0, can be drawn to the graph of y = f(x).

Write down the equation of each tangent.

.....

..... [2]

(b) **(i)** Complete the table for $g(x) = \frac{2}{x} + 3$ for $-3.5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 2.5$.

x	–3.5	–3	–2	–1	–0.5		0.5	1	2	2.5
g(x)	2.4	2.3		1			7	5		3.8

[3]

(ii) On the grid opposite, draw the graph of y = g(x). [4]

(iii) Use your graph to solve the equation f(x) = g(x).

x = ... or x = ...[2]

(c) Find gf(–2).

..... [2]

(d) Find $g^{-1}(5)$.

..... [1]

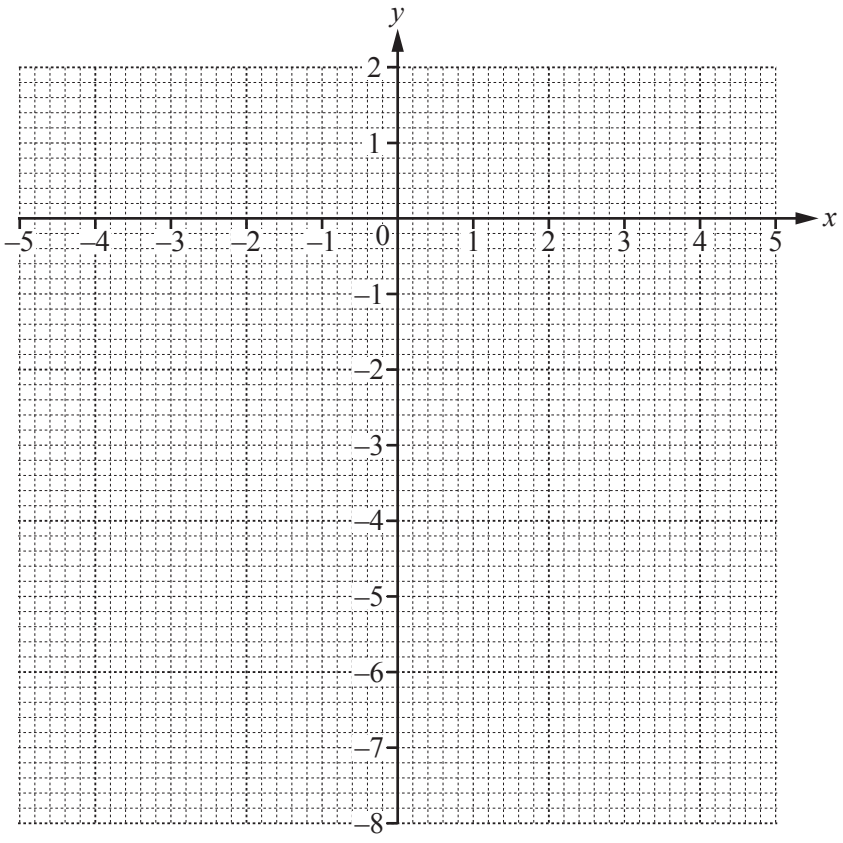
4 $y = 1 - \frac{2}{x^2}, \ x \neq 0$

(a) Complete the table.

x	–5	–4	–3	–2	–1	–0.5		0.5	1	2	3	4	5
y		0.88	0.78			–7		–7			0.78	0.88	

[3]

(b) On the grid, draw the graph of $y = 1 - \frac{2}{x^2}$ for $-5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 5$.



[5]

(c) **(i)** On the grid, draw the graph of $y = -x - 1$ for $-3 \leqslant x \leqslant 5$. [2]

(ii) Solve the equation $1 - \frac{2}{x^2} = -x - 1$.

$x =$ ..... [1]

(iii) The equation $1-\frac{2}{x^2}=-x-1$ can be written in the form $x^3+px^2+q=0$.

Find the value of p and the value of q.

p = ...

q = ... [3]

(d) The graph of $y=1-\frac{2}{x^2}$ cuts the positive x-axis at A.

B is the point $(0, -2)$.

(i) Write down the co-ordinates of A.

(..... , ...) [1]

(ii) On the grid, draw the straight line that passes through A and B. [1]

(iii) Complete the statement.

The straight line that passes through A and B is a .....

at the point ..... [2]

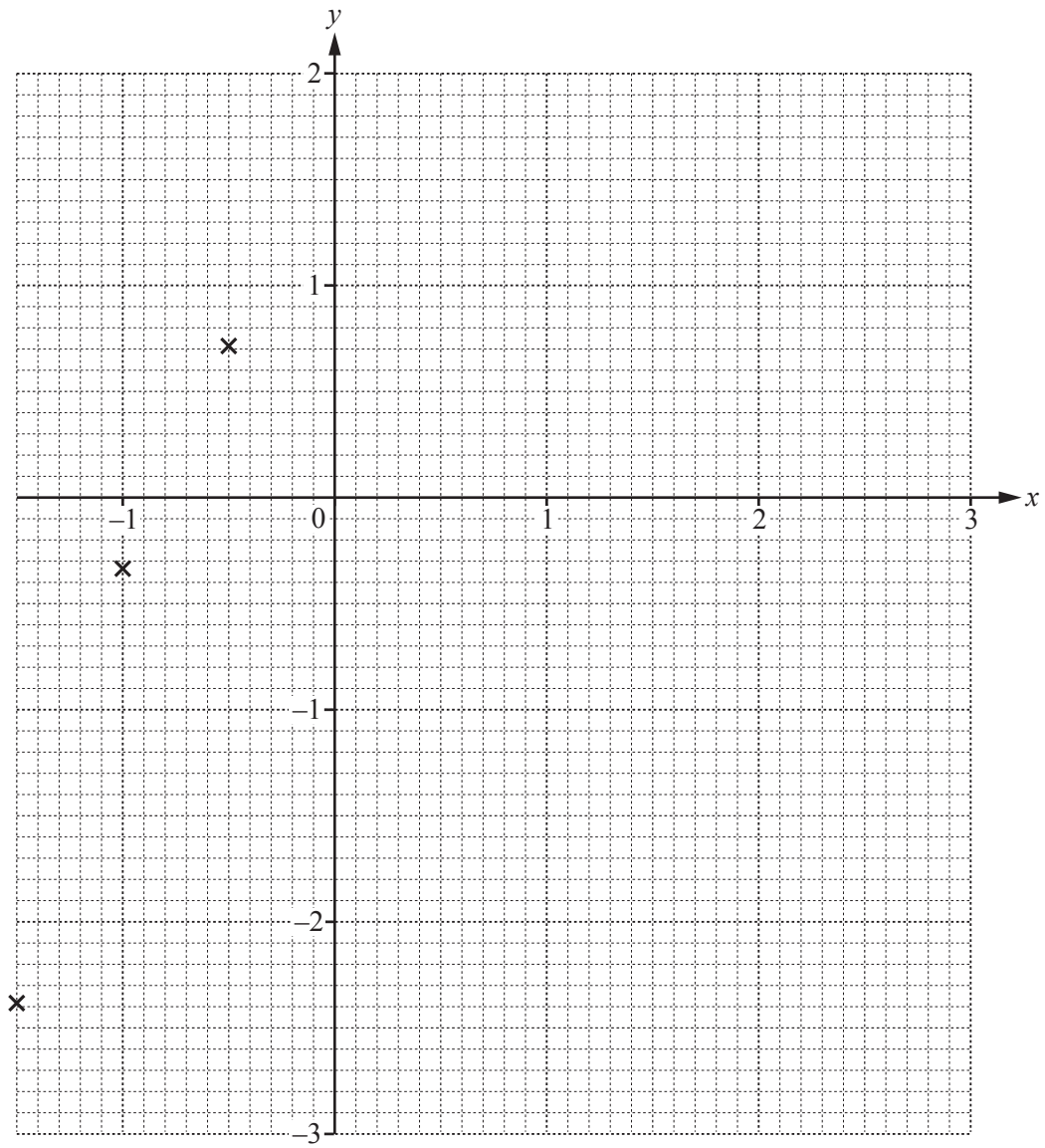
2 **(a)** Complete the table of values for $y = \frac{x^3}{3} - x^2 + 1$.

x	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
y	–2.38	–0.33	0.71		0.79	0.33	–0.13	–0.33	–0.04	

[2]

(b) Draw the graph of $y = \frac{x^3}{3} - x^2 + 1$ for $-1.5 \leqslant x \leqslant 3$.

The first 3 points have been plotted for you.



[4]

(c) Using your graph, solve the equations.

(i) $\frac{x^3}{3} - x^2 + 1 = 0$

$x =$ ..... or $x =$ ..... or $x =$ .....[3]

(ii) $\frac{x^3}{3} - x^2 + x + 1 = 0$

$x =$ ..... [2]

(d) Two tangents to the graph of $y = \frac{x^3}{3} - x^2 + 1$ can be drawn parallel to the x-axis.

(i) Write down the equation of each of these tangents.

.....

..... [2]

(ii) For $0 \leqslant x \leqslant 3$, write down the smallest possible value of y.

$y =$ ..... [1]

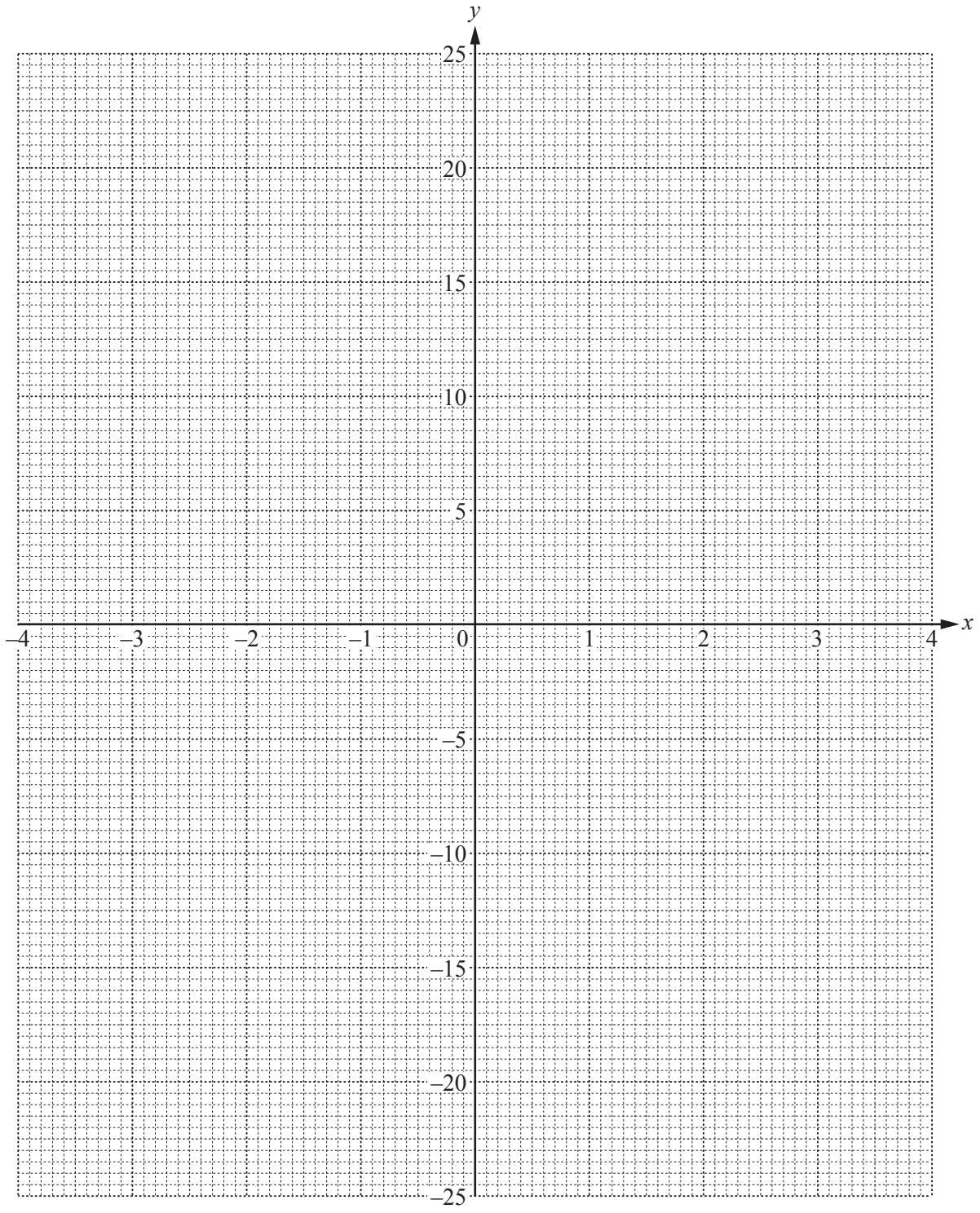
5 $y = x^2 - 2x + \frac{12}{x}, \ x \neq 0$

(a) Complete the table of values.

x	–4	–3	–2	–1	–0.5		0.5	1	2	3	4
y	21	11		–9	–22.75		23.25	11	6		11

[2]

(b) On the grid, draw the graph of $y = x^2 - 2x + \frac{12}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.



[5]

(c) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point (1, 11).

Answer(c) ..... [3]

(d) The equation $x^2 - 2x + \frac{12}{x} = k$ has exactly two distinct solutions.

Use the graph to find

(i) the value of k,

Answer(d)(i) k = ... [1]

(ii) the solutions of $x^2 - 2x + \frac{12}{x} = k$.

Answer(d)(ii) x = ... or x = ... [2]

(e) The equation $x^3 + ax^2 + bx + c = 0$ can be solved by drawing the line $y = 3x + 1$ on the grid.

Find the value of a, the value of b and the value of c.

Answer(e) a = ...

b = ...

c = ... [3]

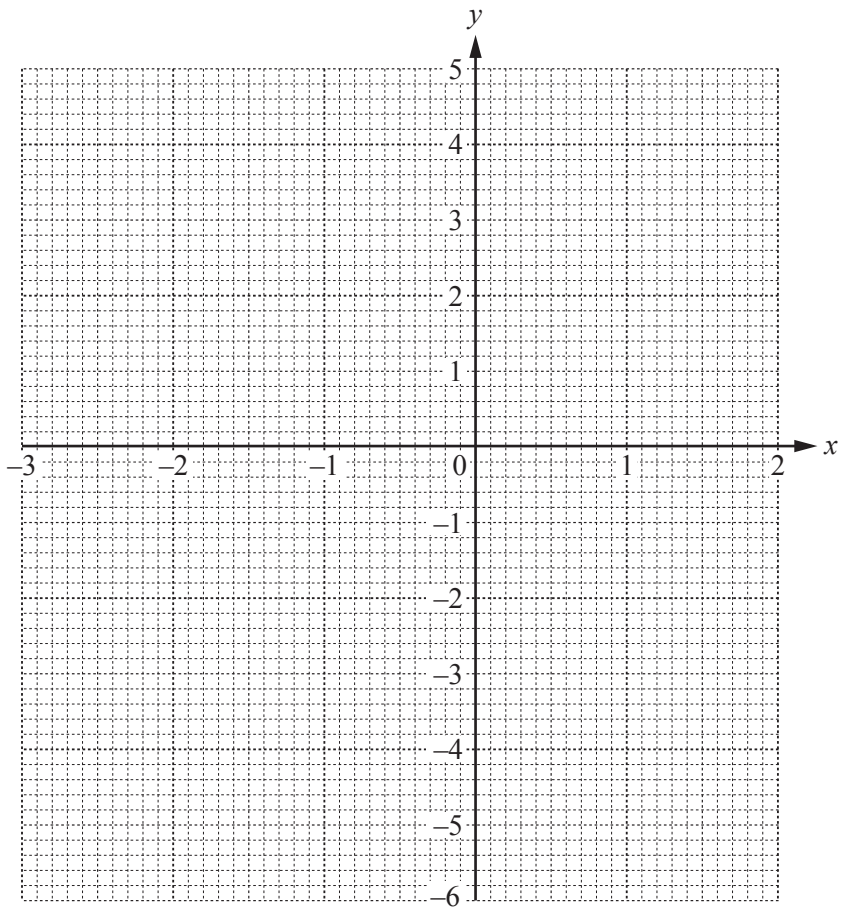
4 $f(x) = x - \frac{1}{2x^2}, \quad x \neq 0$

(a) Complete the table of values.

x	–3	–2	–1.5	–1	–0.5	–0.3		0.3	0.5	1	1.5	2
$f(x)$	–3.1	–2.1	–1.7		–2.5	–5.9		–5.3	–1.5		1.3	1.9

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 2$.



[5]

(c) Use your graph to solve the equation $f(x) = 1$.

Answer(c) $x =$ ..... [1]

(d) There is only one negative integer value, k, for which $f(x) = k$ has only one solution for all real x.

Write down this value of k.

Answer(d) k = ... [1]

(e) The equation $2x - \frac{1}{2x^2} - 2 = 0$ can be solved using the graph of $y = f(x)$ and a straight line graph.

(i) Find the equation of this straight line.

Answer(e)(i) y = ... [1]

(ii) On the grid, draw this straight line and solve the equation $2x - \frac{1}{2x^2} - 2 = 0$.

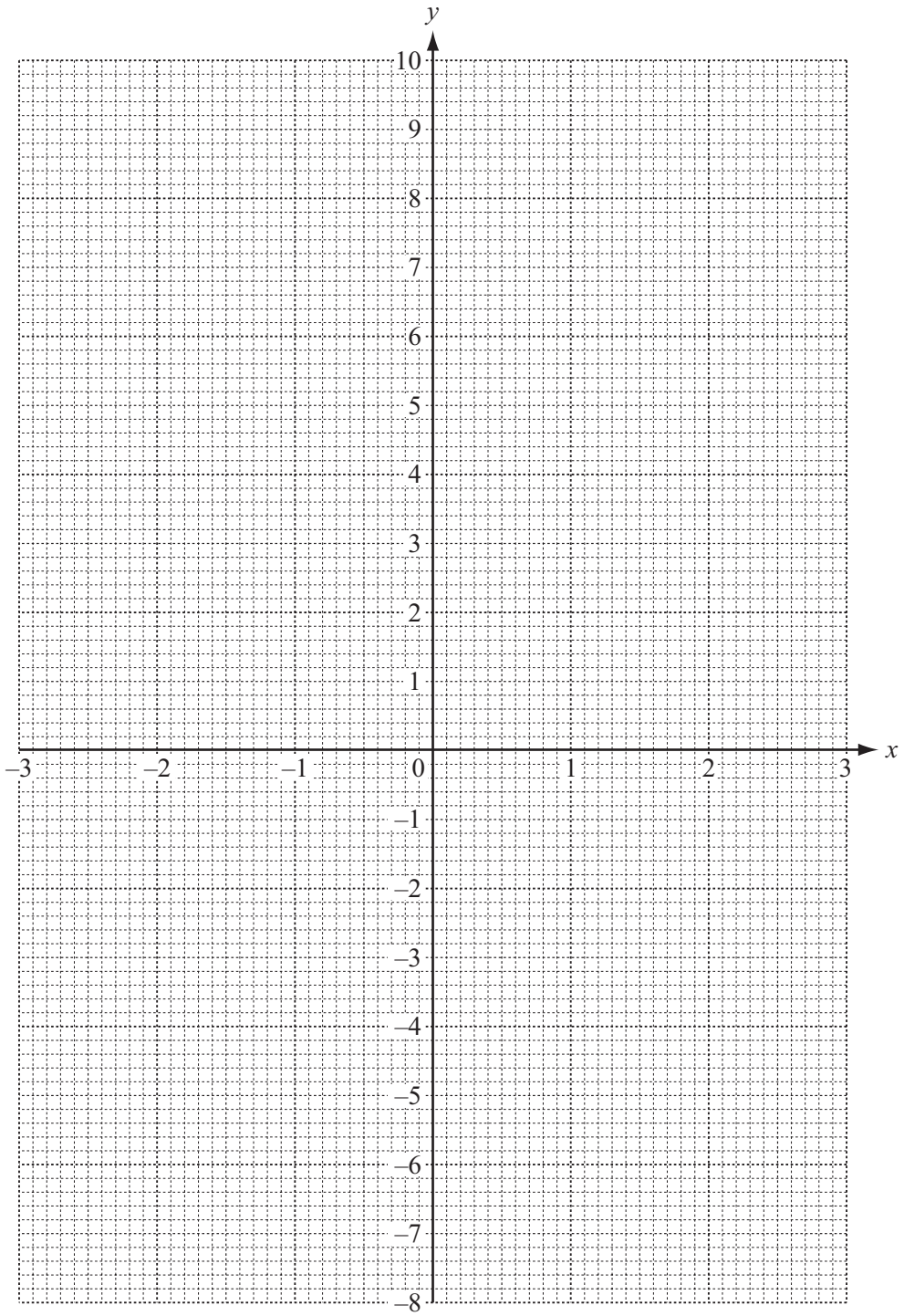
Answer(e)(ii) x = ... [3]

8 **(a)** Complete the table of values for $y = x^3 - 3x + 1$.

x	–2.5	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5
y	–7.125	–1		3		1	–0.375	–1	–0.125	3	9.125

[2]

(b) Draw the graph of $y = x^3 - 3x + 1$ for $-2.5 \leqslant x \leqslant 2.5$.



[4]

(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where $x = 2$.

Answer(c) ..... [3]

(d) Use your graph to solve the equation $x^3 - 3x + 1 = 1$.

Answer(d) $x =$ ..... or $x =$ ..... or $x =$ ..... [2]

(e) Use your graph to complete the inequality in k for which the equation

$x^3 - 3x + 1 = k$ has three different solutions.

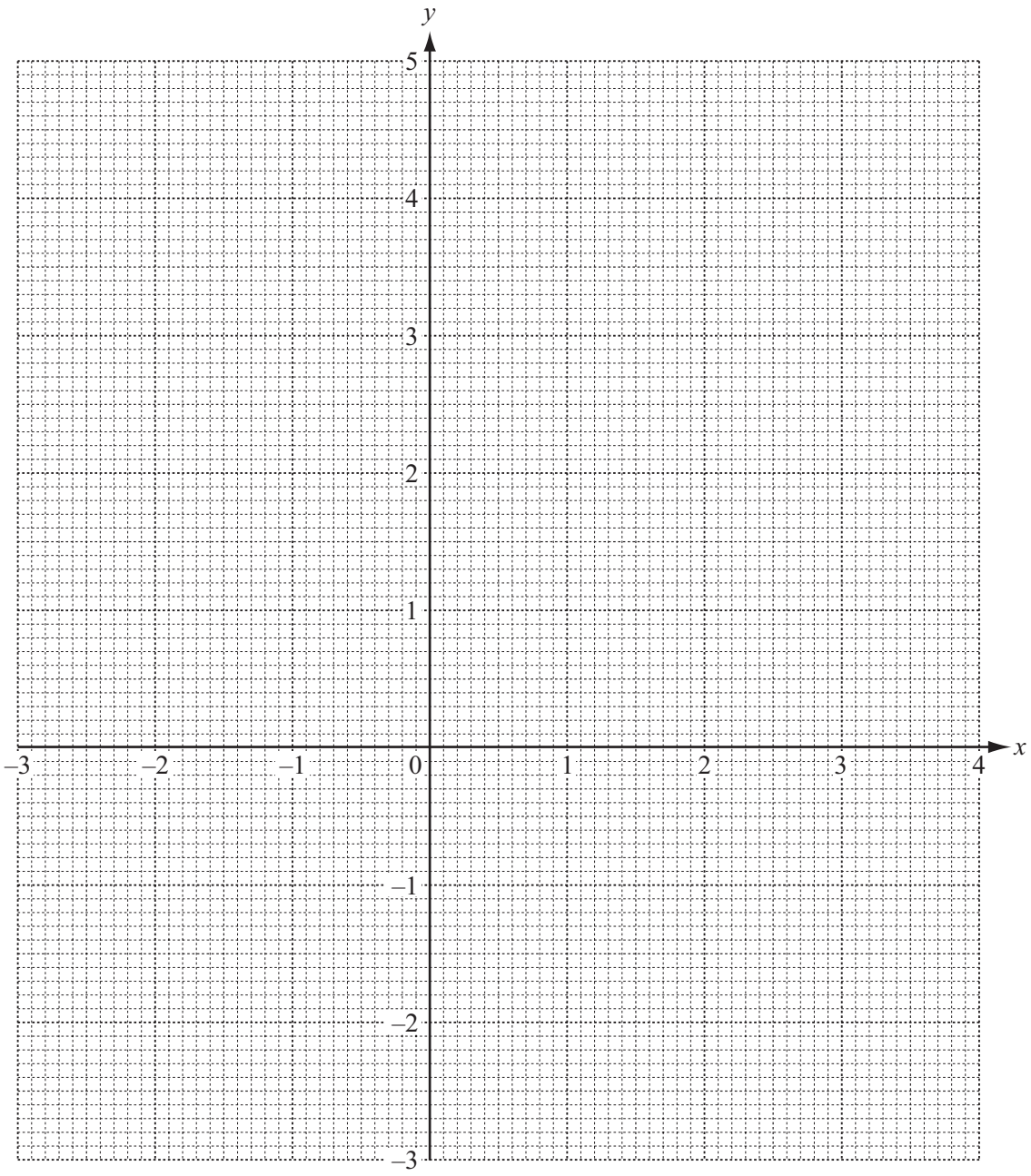
Answer(e) ..... $< k <$ ..... [2]

4 The table shows some values for the function $y = \frac{1}{x^2} + x$, $x \neq 0$.

x	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–2.89	–1.75		3.5		2	2.25		4.06

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = \frac{1}{x^2} + x$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.



[5]

(c) Use your graph to solve the equation $\frac{1}{x^2} + x - 3 = 0$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) Use your graph to solve the equation $\frac{1}{x^2} + x = 1 - x$.

Answer(d) $x =$ ..... [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

Answer(e) ..... [3]

(f) Using algebra, show that you can use the graph at $y = 0$ to find $\sqrt[3]{-1}$.

Answer(f)

[3]

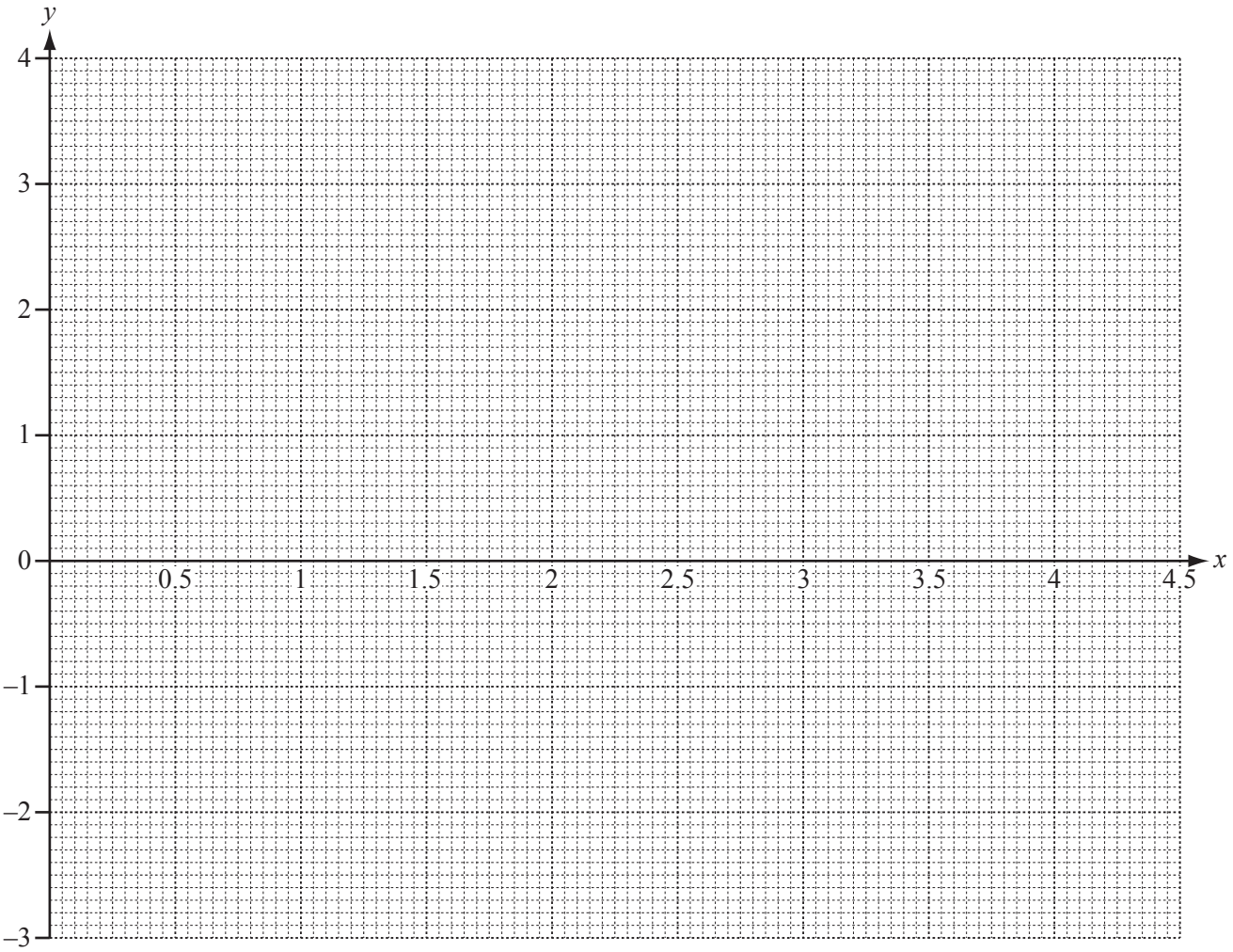
For Examiner's Use

3 The table shows some values for the function $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.

x	1	1.5	2	2.5	3	3.5	4	4.5
y	–3		2	3	3			

(a) Complete the table of values. [3]

(b) On the grid below, draw the graph of $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.



[4]

(c) By drawing a suitable line, use your graph to solve the equation $11x - 2x^2 = 11$.

Answer(c) $x =$ ..... or $x =$ ..... [2]

(d) The line $y = mx + 2$ is a tangent to the curve $y = 11x - 2x^2 - 12$ at the point P.

By drawing this tangent,

(i) find the co-ordinates of the point P,

Answer(d)(i) (..... , ...) [2]

(ii) work out the value of m.

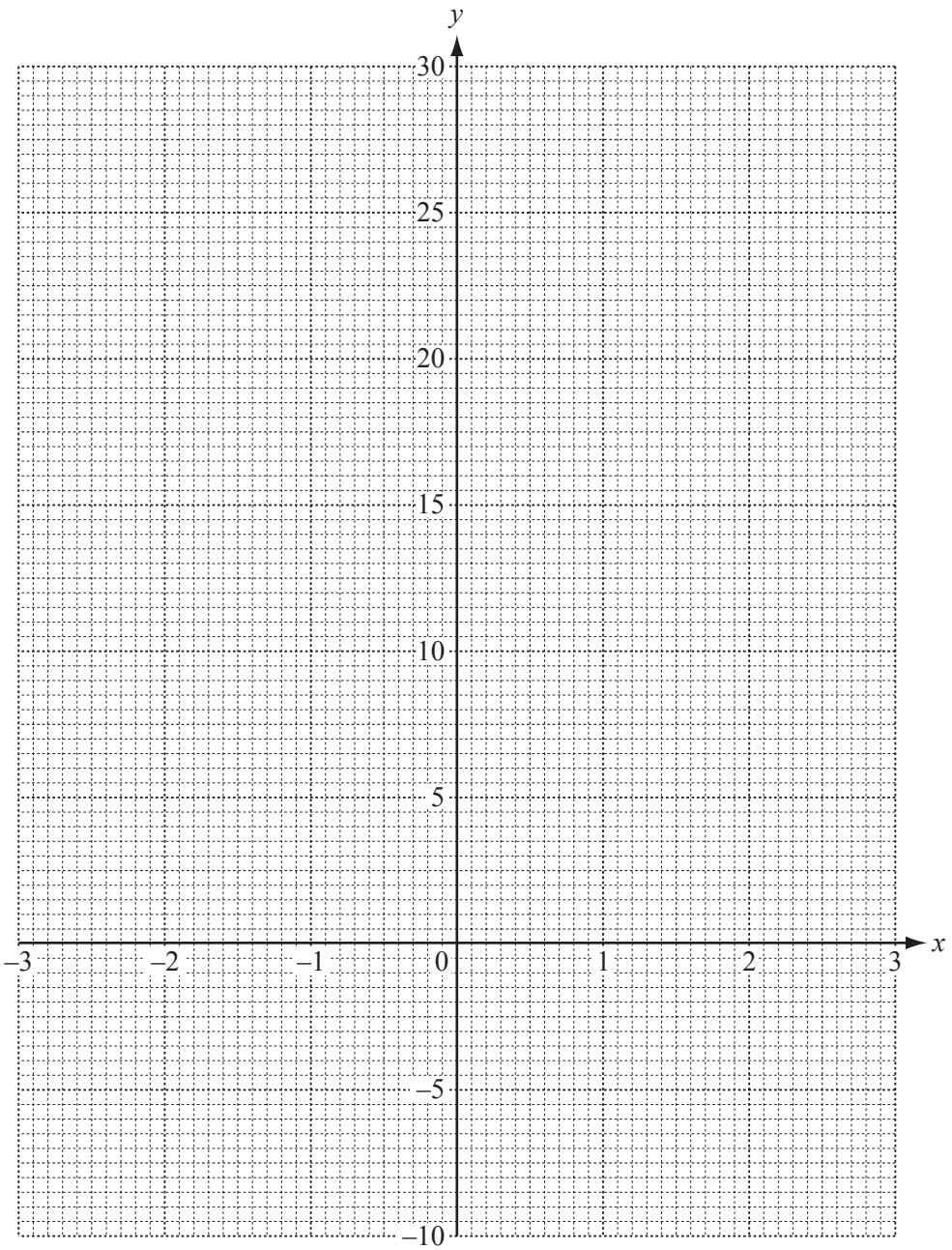
Answer(d)(ii) $m =$ ..... [2]

5 **(a)** Complete the table of values for $y = \frac{2}{x^2} - \frac{1}{x} - 3x$.

x	–3	–2	–1	–0.5	–0.3		0.3	0.5	1	2	3
y	9.6		6		26.5		18.0		–2	–6	–9.1

[3]

(b) Draw the graph of $y = \frac{2}{x^2} - \frac{1}{x} - 3x$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.



[5]

For Examiner's Use

(c) Use your graph to solve these equations.

(i) $\frac{2}{x^2} - \frac{1}{x} - 3x = 0$

Answer(c)(i) x = ... [1]

(ii) $\frac{2}{x^2} - \frac{1}{x} - 3x - 7.5 = 0$

Answer(c)(ii) x = ... or x = ... or x = ... [3]

(d) (i) By drawing a suitable straight line on the graph, solve the equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$.

Answer(d)(i) x = ... or x = ... [4]

(ii) The equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$ can be written in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

Find the values of a, b and c.

Answer(d)(ii) a = ... , b = ... , c = ... [3]